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Effects of covering layer thickness on Love waves in functionally graded piezoelectric substrates

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Abstract An analytical approach is used to investigate the effects of covering layer thickness on the propagation behavior of Love waves in functionally graded piezoelectric materials (FGPMs) covered with a dielectric layer. The piezoelectric substrate is polarized in the direction perpendicular to the wave propagation plane, and its material parameters change continuously along the thickness direction. The dispersion equations for the existence of Love waves with respect to phase velocity are obtained for electrically open and shorted cases, respectively. A detailed investigation of the effects of the covering dielectric layer thickness on dispersion curve, phase velocity, group velocity, and electromechanical coupling factor is carried out. Numerical results show that for a given FGPM, the covering dielectric layer thickness affects significantly the fundamental mode of Love waves but has only negligible effects on the high-order modes. The changes in phase velocity, group velocity, and electromechanical coupling factor due to the change of gradient coefficient of FGPMs could be approached approximately by changing the thickness of the covering dielectric layer, which imply a potential factor for designing new-type surface wave devices with FGPMs.

Keywords Love waves · Functionally graded piezoelectric materials (FGPMs) · Covering layer thickness · Dispersion relation · Electromechanical coupling factor

1 Introduction

Surface acoustic wave devices (such as filters, delay lines, oscillators, and amplifiers) have been widely used in electronic industry for signal transmission, signal processing, and information storage applications since the interdigital transducers (IDT) [1] were successfully utilized for transmitting and receiving surface acoustic waves (such as Love wave). To achieve high performance, many such devices adopt layered piezoelectric structures consisting of a piezoelectric layer and an elastic substrate or an elastic layer and a piezoelectric substrate. Much research work has been done on the propagation of the transverse surface waves in such layered structures [2–9]. Due to the intrinsic brittle property of ceramic materials and the thermal mismatch between layer and substrate, however, initial stresses appear inevitably in such layered structures during the manufacture process and have significant effects on the propagation behavior of the transverse surface waves [10–16].

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A concept that may be used to reduce the magnitude of residual and thermal stresses inevitably appearing in such layered structures could be the introduction of functionally graded piezoelectric materials (FGPMs) [17–21] resulting from the idea of functionally graded materials (FGMs) that are well known to reduce stress concentration near material edges or ends and increase material fracture toughness. Li et al. [17] studied the features of Love waves in a layered functionally graded piezoelectric structure consisting of an FGPM layer and an elastic substrate, where the variations of material constants are independent through the thickness of the layer. Following the work by Li et al. [17], Liu and Wang [18] did a similar work but assumed a different mathematic model for the material gradient of FGPMs in their work. For the same structure, Du et al. [19] and Qian et al. [20,21] separately studied the propagation of Love waves but assumed the same exponential function variation for all material parameters along the thickness direction of the layer. Furthermore, Liu et al. [22] investigated again the Love waves in a smart functionally graded piezoelectric composite structure in which an FGPM layer is placed between a pure piezoelectric material layer and a metal substrate. It can be concluded from those existing work that material gradient could be a potential factor for designing surface acoustic wave devices with FGPMs. However, in practice, the improvement of piezoelectricity is the key to the fabrication of FGPMs. The powder metallurgical processing for fabricating FGPMs is quite complicated at present, and it is still difficult to estimate material property changes synchronously in term of a certain law. Therefore, the material gradient of the manufactured FGPMs is often far from the desired design. And, it's impossible to change the material gradient properties of those manufactured FGPMs, which gives rise to lots of waste. From the point view of economy and energy-saving, we need to look for other design parameters for designing surface wave devices with FGPMs in order to reuse those manufactured FGPMs, such as covering layer thickness. To the best of the authors' knowledge, such work has not been done yet.

In this paper, we will investigate how the thickness of the covering dielectric layer affects the Love wave propagation behavior in a layered structure consisting of an FGPM substrate and a dielectric layer. The statement of the problem is given in Sect. 2. A detailed solution procedure is presented in Sect. 3, followed by numerical examples and discussion in Sect. 4. Some conclusions are drawn in Sect. 5.

2 Statement of the problem

Consider a structure consisting of a homogeneous dielectric layer with uniform thickness of h deposited perfectly on a functionally graded piezoelectric substrate, as shown in Fig. 1. The coordinate system o-xyz is chosen in such way that the z-axis is directed along the poling direction perpendicular to $x - y$ plane, the plane $x = 0$ occupies the interface between the layer and the substrate, and the x-axis points down into the substrate. The domain $x < -h$ is assumed to be vacuum or air, and the surface $x = -h$ is free of external forces (mechanically traction free). The material parameters in the substrate change gradually along the x-axis direction.

Here, Love wave propagation in such a layered functionally graded piezoelectric structure will be taken into account. It is assumed without loss of generality that the wave propagation is in the positive direction of y-axis, such that the mechanical displacement components and electrical potential function representing the

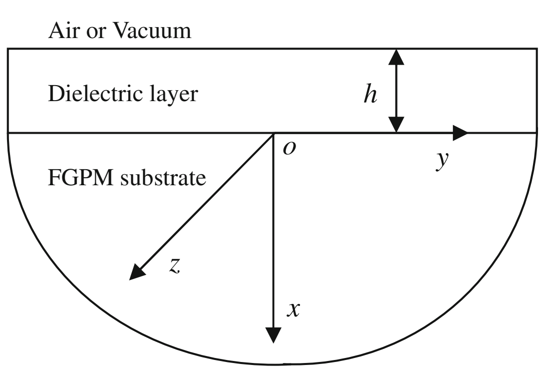

Fig. 1 A schematic configuration of a layered half-space and coordinate system

motion can be written in the following form:

$$\left.\begin{array}{l} u \equiv \upsilon \equiv 0, w = w(x, y, t), \quad -h \leq x \leq +\infty \\ \varphi = \varphi(x, y, t), \quad -\infty \leq x \leq +\infty \end{array}\right\}, \tag{1}$$

where u, υ, and w are the mechanical displacement components along x, y, and z axes, respectively.

For the dielectric layer in the region $-h < x < 0$, let w_1 and φ_1 denote the mechanical displacement and electrical potential function. If the layer is anisotropic, we assume it to have the same anisotropy as the substrate and with axis of polarization parallel to that of the substrate. Otherwise, the layer is assumed to be isotropic. Then, we have the following governing field equations:

$$\left.\begin{array}{l} \nabla^2 w_1 = \frac{1}{c_1^2}\frac{\partial^2 w_1}{\partial t^2} \\ \nabla^2 \varphi_1 = 0 \end{array}\right\}, \tag{2}$$

and the nonzero stress and electric displacement components

$$\left.\begin{array}{l} \tau_{yz}^{(1)} = \mu\frac{\partial w_1}{\partial y}, \quad \tau_{zx}^{(1)} = \mu\frac{\partial w_1}{\partial x} \\ D_x^{(1)} = -\varepsilon\frac{\partial \varphi_1}{\partial x}, \quad D_y^{(1)} = -\varepsilon\frac{\partial \varphi_1}{\partial y} \end{array}\right\}, \tag{3}$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ and $c_1 = (\mu/\rho)^{1/2}$ is the bulk shear wave velocity with μ and ρ being separately the shear modulus and mass density in the layer. ε is the dielectric constant.

In a general transversely isotropic piezoelectric material, the dynamic electromechanical interconnected behavior can be described in a quasi-static approximation by the following governing field equation:

$$\left.\begin{array}{l} \sigma_{ij,i} = \rho\ddot{u}_j \\ D_{i,i} = 0 \end{array}\right\}, \tag{4}$$

and constitutive relations

$$\left.\begin{array}{l} \sigma_{ij} = c_{ijkl}u_{k,l} + e_{lij}\varphi_{,l} \\ D_i = e_{ikl}u_{k,l} - \varepsilon_{il}\varphi_{,l} \end{array}\right\}, \tag{5}$$

where u_j are the mechanical displacement components, φ is the electrical potential, ρ is the mass density of the medium, σ_{ij} and D_i are the stress and electric displacement fields, c_{ijkl}, e_{ikl}, and ε_{il} are the elastic, piezoelectric, and dielectric constants, respectively.

For the functionally graded piezoelectric substrate in the region $x > 0$, let w_2 and φ_2 denote the mechanical displacement and electrical potential function; then, from Eqs. (4) and (5), we have the following coupled electromechanical field equations:

$$\left.\begin{array}{l} \frac{\mathrm{d}c_{44}(x)}{\mathrm{d}x}\frac{\partial w_2}{\partial x} + c_{44}(x)\nabla^2 w_2 + \frac{\mathrm{d}e_{15}(x)}{\mathrm{d}x}\frac{\partial \varphi_2}{\partial x} + e_{15}(x)\nabla^2\varphi_2 = \rho(x)\frac{\partial^2 w_2}{\partial t^2} \\ \frac{\mathrm{d}e_{15}(x)}{\mathrm{d}x}\frac{\partial w_2}{\partial x} + e_{15}(x)\nabla^2 w_2 - \frac{\mathrm{d}\varepsilon_{11}(x)}{\mathrm{d}x}\frac{\partial \varphi_2}{\partial x} - \varepsilon_{11}(x)\nabla^2\varphi_2 = 0 \end{array}\right\}, \tag{6}$$

and the nonzero stress and electric displacement components

$$\left.\begin{array}{l} \tau_{yz}^{(2)} = c_{44}(x)\frac{\partial w_2}{\partial y} + e_{15}(x)\frac{\partial \varphi_2}{\partial y}, \quad \tau_{zx}^{(2)} = c_{44}(x)\frac{\partial w_2}{\partial x} + e_{15}(x)\frac{\partial \varphi_2}{\partial x} \\ D_x^{(2)} = e_{15}(x)\frac{\partial w_2}{\partial x} - \varepsilon_{11}(x)\frac{\partial \varphi_2}{\partial x}, \quad D_y^{(2)} = e_{15}(x)\frac{\partial w_2}{\partial y} - \varepsilon_{11}(x)\frac{\partial \varphi_2}{\partial y} \end{array}\right\}, \tag{7}$$

where $c_{44}(x)$, $e_{15}(x)$, and $\varepsilon_{11}(x)$ are the elastic, piezoelectric, and dielectric constants in the substrate, respectively.

Since our focus in this paper is placed on how the covering layer thickness affects the Love wave propagation behavior, for the simplicity of mathematical handling, we assume that all material parameters in the functionally graded piezoelectric substrate have the same exponential function variation along the x-axis direction by following the previous work [19–21]

$$c_{44}(x) = c_{44}^0 \mathrm{e}^{\alpha x}, \quad e_{15}(x) = e_{15}^0 \mathrm{e}^{\alpha x}, \quad \varepsilon_{11}(x) = \varepsilon_{11}^0 \mathrm{e}^{\alpha x}, \quad \rho(x) = \rho^0 \mathrm{e}^{\alpha x}, \tag{8}$$

where α is the exponential coefficient indicating the profile of the material gradient along the x-axis direction, and the quantities with superscript 0 are the corresponding values of these parameters at the interface $x = 0$.

Substitution of Eq. (8) into Eq. (6) yields

$$\left.\begin{aligned} c_{44}^{0}\left(\alpha \frac{\partial w_2}{\partial x} + \nabla^2 w_2\right) + e_{15}^{0}\left(\alpha \frac{\partial \varphi_2}{\partial x} + \nabla^2 \varphi_2\right) = \rho^{0} \frac{\partial^2 w_2}{\partial t^2} \\ \varepsilon_{11}^{0}\left(\alpha \frac{\partial \varphi_2}{\partial x} + \nabla^2 \varphi_2\right) - e_{15}^{0}\left(\alpha \frac{\partial w_2}{\partial x} + \nabla^2 w_2\right) = 0 \end{aligned}\right\}, \tag{9}$$

which are the wave motion equations for the functionally graded piezoelectric substrate.

Usually, the dielectric constant ε_0 of air is much smaller than that of the dielectric layer and is negligible; thus, the space above the covering layer can be treated as vacuum. Hence, the electrical potential function $\varphi_0(x, y, t)$ in the domain $x < -h$ satisfies the following Laplace's equation:

$$\nabla^2 \varphi_0 = 0 \tag{10}$$

When the Love waves propagate in the layered structure shown in Fig. 1, the mechanical displacement components and the electrical potential function must not only satisfy Eqs. (2), (9) and (10) but also the following boundary conditions and continuity conditions:

1. The mechanical traction-free condition at $x = -h$,

$$\tau_{zx}^{(1)}(-h, y) = 0$$

2. The electrical boundary conditions at $x = -h$,

$$\left.\begin{aligned} \varphi_0(-h, y) = \varphi_1(-h, y) \\ D_x^{(0)}(-h, y) = D_x^{(1)}(-h, y) \end{aligned}\right\} \quad \text{for electrically open case, and}$$

$$\varphi_1(-h, y) = 0 \quad \text{for electrically short case.}$$

3. The continuity conditions at $x = 0$: the normal components of mechanical displacement, stress, electrical potential function, and electric displacement are continuous

$$\left.\begin{aligned} w_1(0, y) = w_2(0, y) \\ \tau_{zx}^{(1)}(0, y) = \tau_{zx}^{(2)}(0, y) \\ \varphi_1(0, y) = \varphi_2(0, y) \\ D_x^{(1)}(0, y) = D_x^{(2)}(0, y) \end{aligned}\right\}.$$

4. For $x \to +\infty$, $w_2 \to 0$, $\varphi_2 \to 0$. For $x \to -\infty$, $\varphi_0 \to 0$.

3 Solution of the problem

3.1 Solutions of the field equations

The solutions of w_1 and φ_1 in the covering dielectric layer can be easily obtained from Eq. (2) as follows:

$$\left.\begin{aligned} w_1(x, y, t) = \left(A_1 e^{ikb_1 x} + A_2 e^{-ikb_1 x}\right) \exp[ik(y - ct)] \\ \varphi_1(x, y, t) = \left(A_3 e^{kx} + A_4 e^{-kx}\right) \exp[ik(y - ct)] \end{aligned}\right\} \tag{11}$$

where A_1, A_2, A_3, and A_4 are arbitrary constants, $k(= 2\pi/\lambda)$ is the wave number with λ being the wavelength, $i = \sqrt{-1}$, c is the phase velocity of wave propagation, $b_1 = (c^2/c_1^2 - 1)^{1/2}$ with c_1 being the bulk shear wave velocity in the layer. The derivation of Eq. (11) is under the assumption $c_1 < c$, the existence condition of Love waves for the layer.

The solutions of the mechanical displacement and electrical potential function in the substrate can be assumed as following form:

$$\left.\begin{aligned} w_2(x, y, t) = W_2(x) \exp[ik(y - ct)] \\ \varphi_2(x, y, t) = \Phi_2(x) \exp[ik(y - ct)] \end{aligned}\right\} \tag{12}$$

where $W_2(x)$ and $\Phi_2(x)$ are the functions to be determined. Substitution of Eq. (12) into Eq. (9) yields

$$\left.\begin{aligned} c_{44}^0\left(W_2''+\alpha W_2'-k^2W_2\right)+e_{15}^0\left(\Phi_2''+\alpha\Phi_2'-k^2\Phi_2\right)=-\rho^0k^2c^2W_2 \\ \varepsilon_{11}^0\left(\Phi_2''+\alpha\Phi_2'-k^2\Phi_2\right)-e_{15}^0\left(W_2''+\alpha W_2'-k^2W_2\right)=0 \end{aligned}\right\} \tag{13}$$

where the prime on W and φ denotes differentiation with respect to x coordinate.

From the second expression in Eq. (13), we have

$$\left(\Phi_2''+\alpha\Phi_2'-k^2\Phi_2\right)=\frac{e_{15}^0}{\varepsilon_{11}^0}\left(W_2''+\alpha W_2'-k^2W_2\right). \tag{14}$$

Substitution of Eq. (14) into the first expression of Eq. (13) gives rise to

$$W_2''+\alpha W_2'+\left(\frac{c^2}{c_2^2}-1\right)k^2W_2=0, \tag{15}$$

where $c_2=\left[\left(c_{44}^0+e_{15}^{02}/\varepsilon_{11}^0\right)/\rho^0\right]^{1/2}$ is the bulk shear wave velocity in the substrate. Under the existence condition of Love waves for the substrate $c<c_2$, considering the boundary condition 4), we can easily obtain the solution of W_2 from Eq. (15) and then get the solution of the mechanical displacement in the substrate from Eq. (12) as follows:

$$w_2(x,y,t)=A_5\mathrm{e}^{rx}\exp\left[\mathrm{i}k\left(y-ct\right)\right] \tag{16}$$

where $r=-\left[\alpha/2+\left(\alpha^2/4+b_2^2k^2\right)^{1/2}\right]$ with $b_2=\left(1-c^2/c_2^2\right)^{1/2}$ and A_5 is an arbitrary constant.

Equation (14) can be regarded as an inhomogeneous equation with respect to Φ_2, so the complete solution of Φ_2 should be the superposition of the homogeneous solution Φ_2^h corresponding to the homogeneous equation of Eq. (14) and the particular solution Φ_2^p of Eq. (14). And obviously, $\Phi_2^P=\left(e_{15}^0/\varepsilon_{11}^0\right)\ W_2$ is a particular solution of Eq. (14). Similarly, considering the existence condition of the Love waves and the boundary condition 4), we can obtain the complete solution of Φ_2 from Eq. (14) and then get the solution of the electrical potential function in the substrate from Eq. (12) as follows:

$$\varphi_2(x,y,t)=\left(A_6\mathrm{e}^{sx}+\frac{e_{15}^0}{\varepsilon_{11}^0}A_5\mathrm{e}^{rx}\right)\exp\left[\mathrm{i}k\left(y-ct\right)\right], \tag{17}$$

where $s=-[\alpha/2+(\alpha^2/4+k^2)^{1/2}]$ and A_6 is an arbitrary constant.

Considering the boundary condition 4), the solution of φ_0 in Eq. (10) can be easily obtained as follows:

$$\varphi_0=A_0\mathrm{e}^{kx}\exp\left[\mathrm{i}k\left(y-ct\right)\right], \tag{18}$$

where A_0 is an arbitrary constant.

3.2 Solutions of the dispersion equations

3.2.1 Electrically open case

Substituting Eq. (11) into Eq. (3), we can obtain the shear stress and the electric displacement in the layer as follows:

$$\left.\begin{aligned} \tau_{zx}^{(1)}&=\mu\left(A_1\mathrm{e}^{\mathrm{i}kb_1x}-A_2\mathrm{e}^{-\mathrm{i}kb_1x}\right)\mathrm{i}kb_1\exp\left[\mathrm{i}k\left(y-ct\right)\right] \\ \tau_{yz}^{(1)}&=\mu\left(A_1\mathrm{e}^{\mathrm{i}kb_1x}+A_2\mathrm{e}^{-\mathrm{i}kb_1x}\right)\mathrm{i}k\exp\left[\mathrm{i}k\left(y-ct\right)\right] \\ D_x^{(1)}&=-\varepsilon k\left(A_3\mathrm{e}^{kx}-A_4\mathrm{e}^{-kx}\right)\exp\left[\mathrm{i}k\left(y-ct\right)\right] \end{aligned}\right\}. \tag{19}$$

Similarly, the shear stress and the electric displacement in the substrate can be obtained by substituting Eqs. (16) and (17) into Eq. (7)

$$\left.\begin{aligned}\tau_{zx}^{(2)} &= \left[P^0 e^{(\alpha+r)x} r A_5 + e_{15}^0 e^{(\alpha+s)x} s A_6\right] \exp\left[ik\left(y-ct\right)\right]\\ \tau_{yz}^{(2)} &= \left[P^0 e^{(\alpha+r)x} A_5 + e_{15}^0 e^{(\alpha+s)x} A_6\right] ik \exp\left[ik\left(y-ct\right)\right]\\ D_x^{(2)} &= -\varepsilon_{11}^0 e^{(\alpha+s)x} s A_6 \exp\left[ik\left(y-ct\right)\right]\end{aligned}\right\}, \tag{20}$$

in which $P^0 = c_{44}^0 + e_{15}^{0\,2}/\varepsilon_{11}^0$ is the piezoelectrically stiffened shear modulus of the substrate.

And the electric displacement in the air is

$$D_x^{(0)} = -\varepsilon_0 A_0 e^{kx} \exp\left[ik\left(y-ct\right)\right]. \tag{21}$$

Substitution of Eqs. (11), (16–17) and (19–21) into the boundary conditions 1) and 2) and continuity condition 3) yields the following homogeneous linear algebraic equations with respect to A_0, A_1, A_2, A_3, A_4, A_5, and A_6

$$\left.\begin{aligned}&A_1 e^{-ikb_1h} - A_2 e^{ikb_1h} = 0\\ &A_3 e^{-kh} + A_4 e^{kh} - A_0 e^{-kh} = 0\\ &(A_3 e^{-kh} - A_4 e^{kh})\varepsilon - \varepsilon_0 A_0 e^{-kh} = 0\\ &A_1 + A_2 - A_5 = 0\\ &A_3 + A_4 - (e_{15}^0/\varepsilon_{11}^0)A_5 - A_6 = 0\\ &(A_1 - A_2) i\mu k b_1 - P^0 r A_5 - e_{15}^0 s A_6 = 0\\ &(A_3 - A_4)\varepsilon k - \varepsilon_{11}^0 s A_6 = 0\end{aligned}\right\}. \tag{22}$$

The non-trivial solution of Eq. (22) exists if and only if the determinant of the coefficient matrix equals zero, which gives rise to the following dispersion equation for the electrically open case:

$$\left[\frac{\varepsilon_{11}^0}{\varepsilon}\frac{\tanh(kh) + \varepsilon/\varepsilon_0}{1 + \tanh(kh)\varepsilon/\varepsilon_0} - \frac{k}{s}\right]\left[\mu b_1 \tan(khb_1) + P^0 \frac{r}{k}\right] + \frac{e_{15}^{0\;2}}{\varepsilon_{11}^0} = 0. \tag{23}$$

3.2.2 *Electrically short circuit*

For the electrically short case (the free surface is plated with a very thin metal strip), the second and third equations in Eq. (22) should be replaced by the following equation which corresponds to the condition (2 in Sect. 2

$$A_3 e^{-kh} + A_4 e^{kh} = 0. \tag{24}$$

Then, a set of homogeneous linear algebraic equations with respect to A_1, A_2, A_3, A_4, A_5, and A_6 can be obtained. By the similar procedure to the electrically open case, we can obtain the corresponding phase velocity equation for the electrically short case

$$\left[\frac{\varepsilon_{11}^0}{\varepsilon}\tanh(kh) - \frac{k}{s}\right]\left[\mu b_1 \tan(khb_1) + P^0 \frac{r}{k}\right] + \frac{e_{15}^{0\;2}}{\varepsilon_{11}^0} = 0. \tag{25}$$

As a matter of fact, the phase velocity equation for the electrically short case has the same form as that for the electrically open case since Eq. (25) can be obtained directly through eliminating the terms with ε_0 in Eq. (23).

Equations (23) and (25) are the phase velocity equations of Love wave propagation in the layered functionally graded piezoelectric structure for the electrically open and short cases, respectively. It is readily seen that the phase velocity c is related to the gradient coefficient, wavelength, covering layer thickness, elastic, dielectric, and piezoelectric constants. The effect of the covering layer thickness on the propagation behavior of Love waves will be discussed detailedly in Sect. 4.

Table 1 Material properties used in computational analysis

Materials	Elastic constant $c_{44}(10^{10}\text{N/m}^2)$	Mass density $\rho(10^3\text{kg/m}^3)$	Piezoelectric constant $e_{15}(\text{c/m}^2)$	Dielectric constant ε_{11} (F/m)
Pb glass (layer)	2.18	3.879	0.0	$5.1\varepsilon_0$
ZnO (substrate)	4.23exp (αx)	5.665exp (αx)	−0.48exp (αx)	$7.57\varepsilon_0$exp (αx)

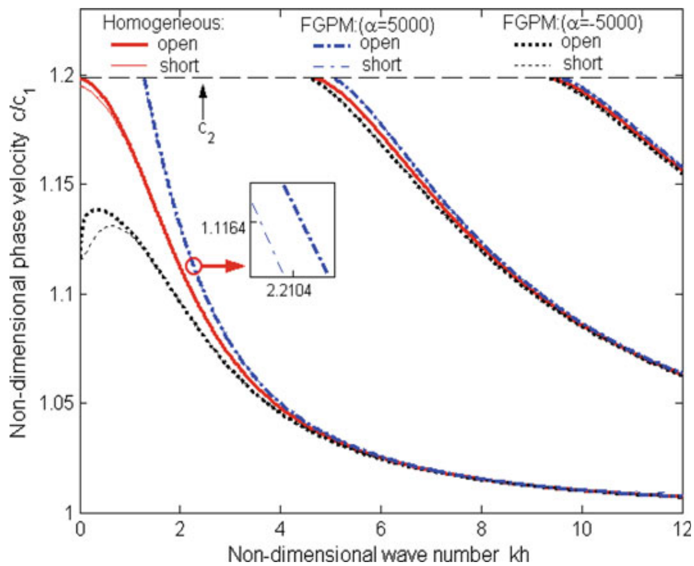

Fig. 2 Dispersion curves of Pb glass layer/ZnO substrate system with/without material gradient

4 Numerical examples

Up to now, the dispersion equations for the existence of Love waves in the layered functionally graded piezoelectric structure have been analytically obtained for electrically open and short cases, respectively. To graphically show the phenomena stated in the abstract, i.e., the changes in phase velocity, group velocity and electromechanical coupling factor induced by the change of gradient coefficient of FGPMs could be approached approximately by properly changing the thickness of the covering dielectric layer, we first calculate the corresponding quantities from the dispersion equations for different selected gradient coefficient values, then calculate the same quantities at a fixed gradient coefficient for different covering layer thickness values, finally compare the calculated results of the two cases. The material parameters [21] used in our calculation are summarized in Table 1. The dielectric constant of vacuum is given by $\varepsilon_0 = 8.85 \times 10^{-12}$F/m.

4.1 Dispersion curve

Based on Eqs. (23) and (25), the dispersion curves for homogeneous substrate and gradient substrate are shown in Fig.2 simultaneously for both the electrically open and short cases, respectively. The effects of gradient coefficient on the first three modes of Love waves are calculated and shown.

It can be seen from Fig. 2 that the effect of the material gradient on the fundamental mode is more obvious than that on the high-order modes. Furthermore, the effect of material gradient on dispersion curves is sensitive to the sign of the gradient coefficient. The plus gradient coefficient has obvious effect on the cut-off frequency for each mode of Love waves, especially pronounced for the fundamental mode, while the minus gradient coefficient has no effect on the cut-off frequency but decreases the phase velocity of the fundamental mode obviously. Through the comparison between the dispersion curves of the two different electrical boundary conditions, it is observed from Fig. 2 that the dispersion curves for the electrically open case resemble those for electrically short case, no matter whether the substrate is functionally graded or homogeneous. Hence, in the following demonstration, only the electrically open case is taken into account. And the detailed discussions will be focused on the fundamental mode where the significant influence is observed in Fig. 2.

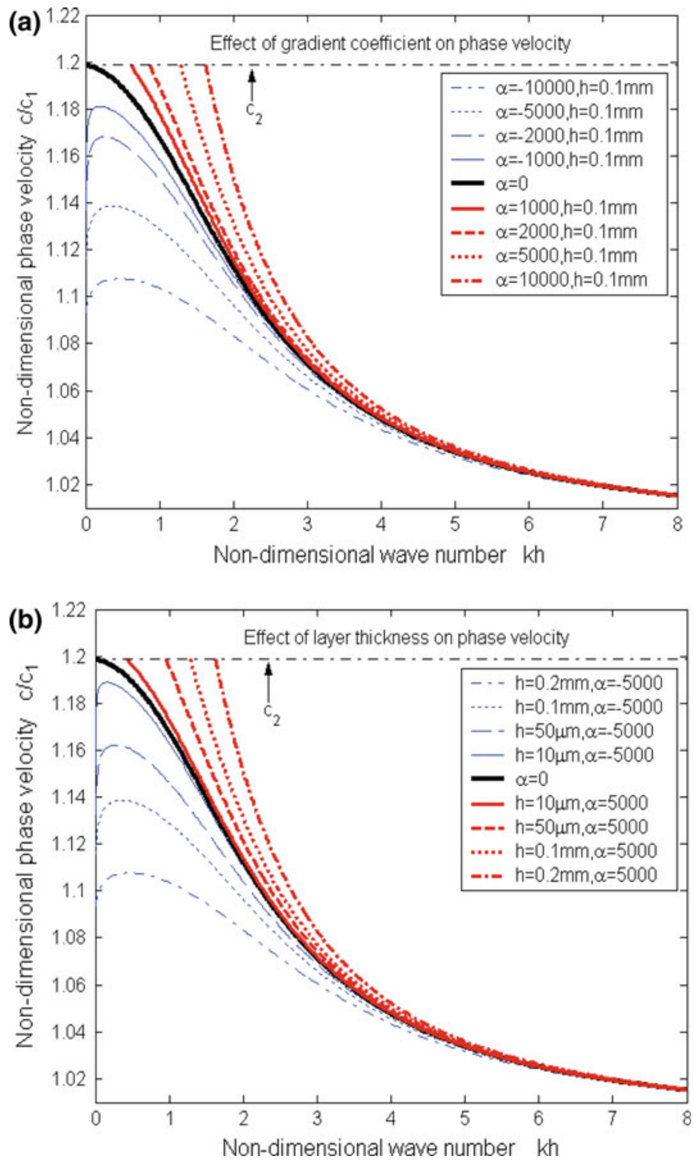

Fig. 3 Phase velocity of the fundamental mode affected by (**a**) material gradient, (**b**) covering layer thickness

It should be noted that the phase velocity of the fundamental mode corresponding to positive material gradient coefficient exceeds the genuine Love wave speed limit c_2 when the non-dimensional wave number tends to zero. For the sake of brevity and clear comparison, that part is not shown in Fig. 2 and all the figures following.

4.2 Phase velocity

Variations of the dispersion curves for selected values of gradient coefficients with a fixed covering layer thickness are shown in Fig. 3a. It can be noted from Fig. 3a that the dispersion curves move to the one of homogeneous substrate (i.e., $\alpha = 0$) when the plus gradient coefficient decreases or the magnitude of the minus gradient coefficient decreases. The results in Fig. 3a show that the more inhomogeneous in the substrate, the more obvious influence the material gradient has on the phase velocity.

Figure 3b shows the similar plots but for selected values of the covering layer thickness with a fixed material gradient coefficient. It can be seen readily from Fig. 3b that for certain plus gradient coefficient, the

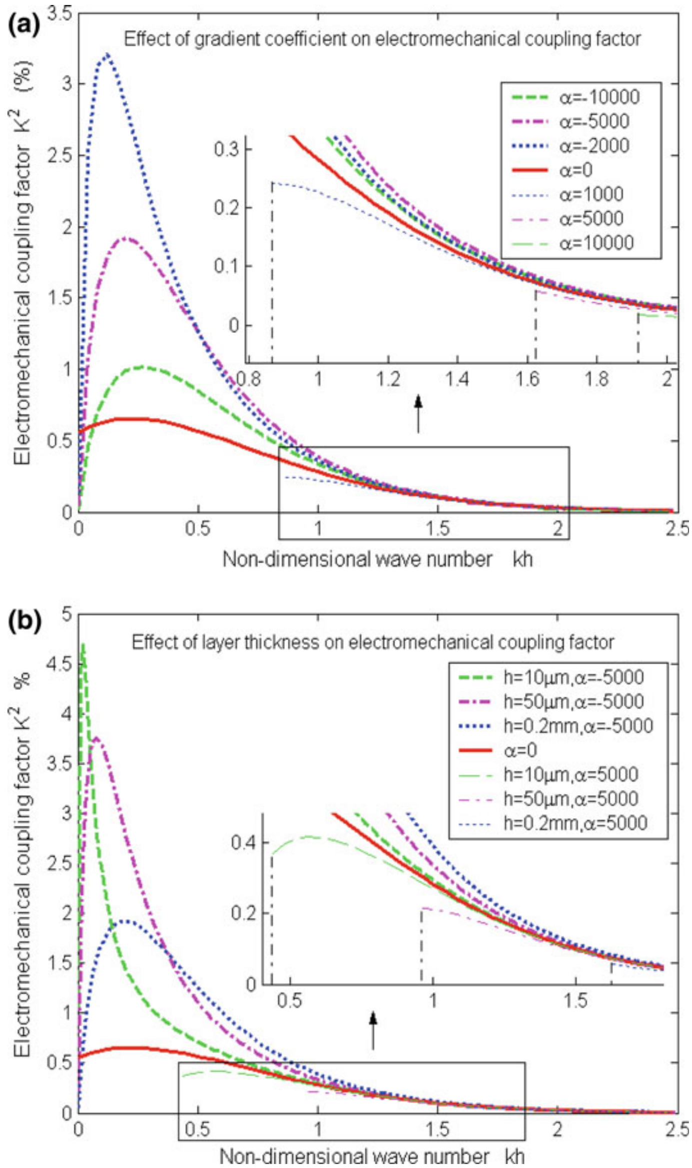

Fig. 4 Electromechanical coupling factor of the fundamental mode affected by (**a**) material gradient, (**b**) covering layer thickness

dispersion curves move to the one of homogeneous substrates (i.e., $\alpha = 0$) with the decrease in the covering layer thickness (as denoted by thick lines in Fig. 3b), and the phenomenon applies to the case of certain minus gradient coefficient (as denoted by thin lines in Fig. 3b). Through the comparison between Fig. 3(a) and Fig. 3b, it is concluded that adjusting the covering layer thickness could also obtain the same effect on the phase velocity/dispersion curves as adjusting the gradient coefficient, which means that the efficiency of SAW devices with the use of FGPM in the substrates can be improved by adjusting the covering layer thickness as well. And this aspect seems more reasonable and executable during the process of designing an SAW device with FGPMs.

Meanwhile, during our calculation, it is found that for the case of $\alpha = 0$, i.e., no gradient in the substrate, the covering layer thickness has a negligible influence on the phase velocity, which is not of practical interest and not shown in detail here.

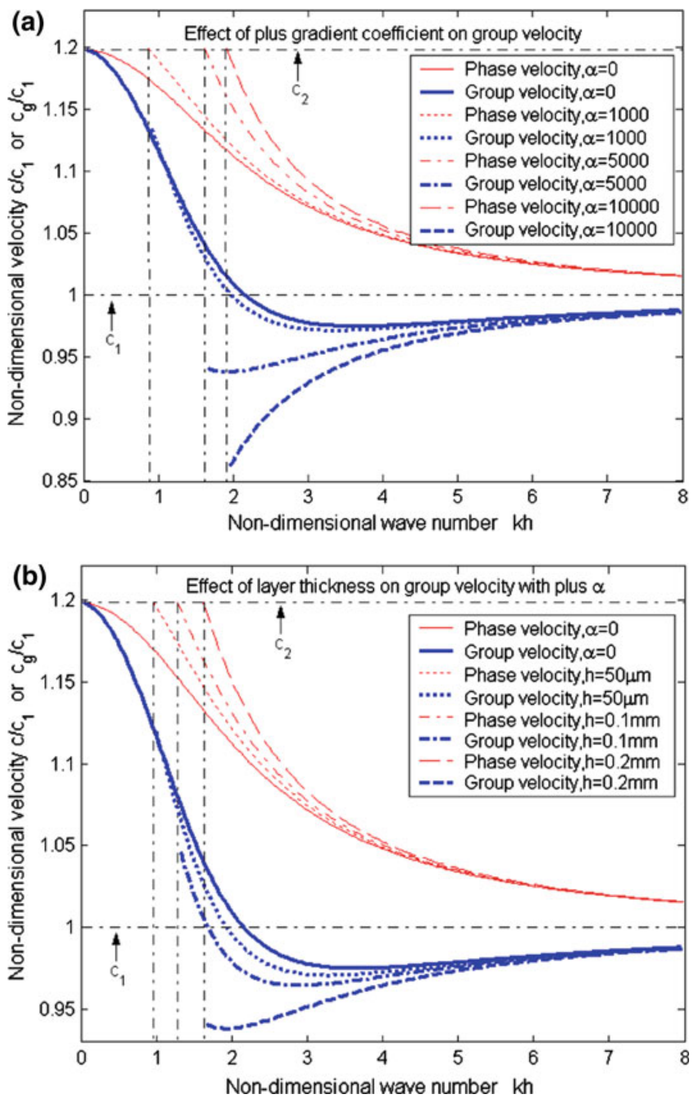

Fig. 5 Group velocity of the fundamental mode affected by (**a**) material gradient, (**b**) covering layer thickness

4.3 Electromechanical coupling factor

Electromechanical coupling factor is an important parameter for designing SAW devices in engineering applications, so we discuss the effects of material gradient and covering layer thickness on this parameter in this Subsection. The electromechanical coupling factor of the mth mode is defined as [23]

$$K_{pm}^2 = \frac{2\left|c_{pm(\text{open})} - c_{pm(\text{short})}\right|}{c_{pm(\text{open})}} \tag{26}$$

where $c_{pm(\text{open})}$ and $c_{pm(\text{short})}$ are phase velocities of the mth mode for the electrically open and short cases, respectively.

Figure 4a shows the electromechanical coupling factor calculated for some selected values of the material gradient coefficient with a fixed covering layer thickness $h = 0.2\,\text{mm}$. It can be seen from Fig. 4a that the minus gradient coefficient increases the electromechanical coupling factor greatly while the plus gradient coefficient decreases a little bit the electromechanical coupling factor. The similar phenomena can be observed in Fig. 4b which is the electromechanical coupling factor calculated for some selected values of the covering layer thickness with a fixed material gradient coefficient. For a fixed minus material gradient coefficient, the change of the covering layer thickness affects the electromechanical coupling factor greatly, while for a fixed

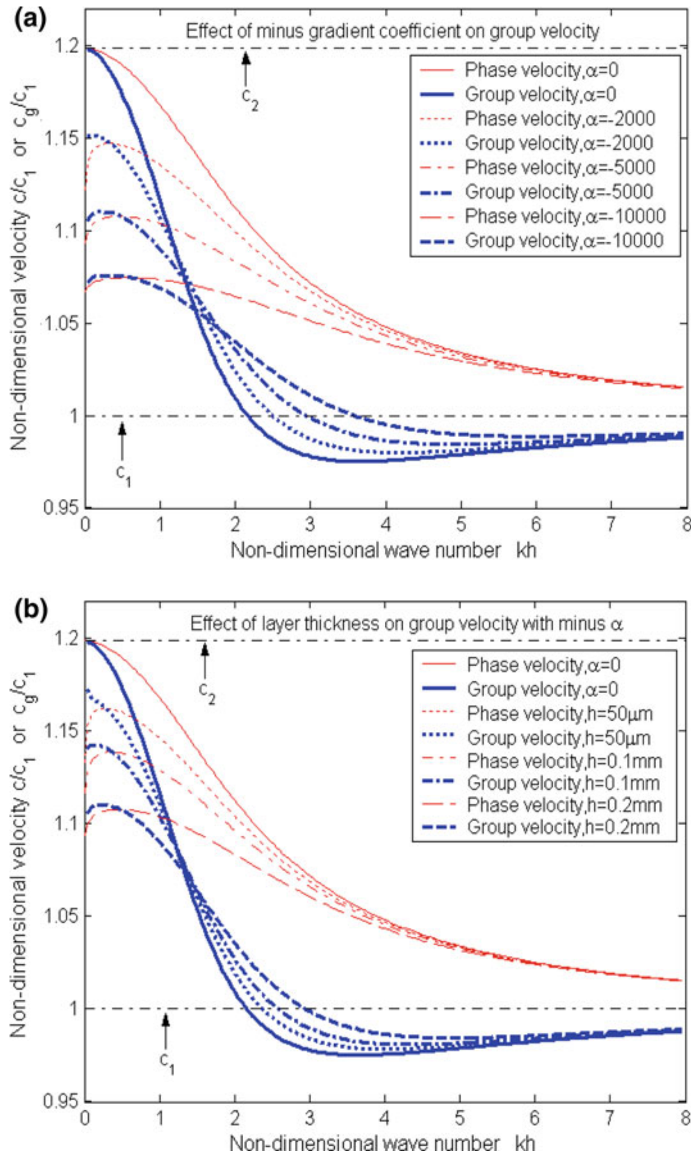

Fig. 6 The same as Fig. 5 but for minus values of material gradient coefficient

plus material gradient coefficient, the change of the covering layer thickness can only give a little bit effect on the electromechanical coupling factor. Through the comparison between Fig. 4a and b, it is concluded that the effect of the material gradient on the electromechanical coupling factor can be approximately approached by properly changing the covering layer thickness at a fixed material gradient coefficient. For some gradient coefficient, the thinner the covering layer is, the higher electromechanical coupling factor you can get, which is pretty useful for the design of SAW devices.

4.4 Group velocity

It is well known that group velocity expresses the rate at which vibration energy is transported. To demonstrate the dispersion behavior of the Love waves further, we study in this subsection the group velocity c_g defined as [24]

$$c_g = c + k\frac{\mathrm{d}c}{\mathrm{d}k} \tag{27}$$

Figure 5a shows the effect of the plus gradient coefficient on the group velocity at a fixed covering layer thickness. It can be seen from Fig. 5a that the group velocity starts with certain value, and as the wave number increases, the group velocity decreases and tends to c_1 at high wave numbers. This result is quite consistent with Eq. (27), because the derivative $\mathrm{d}c/\mathrm{d}k$ tends to zero at high wave numbers, that is to say the group velocity c_g tends to the phase velocity c which is c_1 at high wave numbers. This observation validates our calculations correct to some extent. Similar phenomenon can be observed in Fig. 5b which is plotted for the effect of the covering layer thickness on the group velocity at a fixed plus material gradient coefficient. The comparison between Fig. 5a and b shows that the layer thickness has almost the same effect on the group velocity as the gradient coefficient does. For example, the change in the group velocity made by adjusting the gradient coefficient can also be obtained by adjusting the covering layer thickness with one certain gradient coefficient. The same observation is noticed in Fig. 6a, b which are the same plots as Fig. 5 but for the minus material gradient coefficient.

But when the material gradient disappears, i.e., the gradient coefficient $\alpha = 0$, the layer thickness has a neglectable effect on the group velocity, which agrees well with the result of the layer thickness on the phase velocity shown in Subsection 4.2. This is an obvious result if we pay attention to Eq. (27) from which we can see that the group velocity is only dominated by the phase velocity. Hence, if the layer thickness has no effect on the phase velocity at $\alpha = 0$, it is a matter of course that it has no effect on the group velocity.

5 Conclusions

In this paper, we studied the effect of the covering layer thickness on the Love wave propagation in a layered structure consisting of a functionally graded piezoelectric substrate and a dielectric covering layer. This work is successive to our previous work in which the Love wave propagation in the same structure was investigated, but the focus was placed on the effect of the material gradient. Through the comparison between the effects of the material gradient and the covering layer thickness on the phase velocity, the electromechanical coupling factor, and the group velocity of the Love wave propagation, we can conclude that the changes in phase velocity, group velocity, and electromechanical coupling factor induced by the change of gradient coefficient of FGPMs could be approached approximately by properly adjusting the thickness of the covering dielectric layer, which implies a potential factor for designing new-type surface wave devices with FGPMs. For simplicity, in the present work, we introduce an assumption that all the material parameters have the same exponential function variation of the thickness coordinate. To apply the results obtained here to design high-efficiency SAW devices in practice, further studies on FGPMs of various material gradients are still needed. Furthermore, the assumption that the covering layer is of uniform thickness is adopted in our model, which implies that covering layer of non-uniform thickness may also affect the Love wave propagation significantly or even more significantly than the case of uniform thickness. Those will be our research work in the next step.

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References

1. White, R.M., Voltmer, F.W.: Direct piezoelectric coupling to surface elastic waves. Appl. Phys. Lett **7**, 314–316 (1965)
2. Curtis, R.G., Redwood, M.: Transverse surface waves on a piezoelectric material carrying a metal layer of finite thickness. J. Appl. Phys. **44**, 2002–2007 (1973)
3. Wang, Q., Quek, S.T., Varadan, V.K: Love waves in piezoelectric coupled solid media. Smart Mater. Struct. **10**, 380–388 (2001)
4. Jin, F., Wang, Z.K., Wang, T.J.: The Bleustein-Gulyaev (B-G) wave in a piezoelectric layered half-space. Int. J. Eng. Sci. **39**, 1271–1285 (2001)
5. Yang, J.S.: Love waves in piezoelectromagnetic materials. Acta Mech. **168**, 111–117 (2004)
6. Yang, J.S., Zhou, H.: Acoustoelectric amplification of piezoelectric surface waves. Acta Mech. **172**, 113–122 (2004)
7. Danoyan, Z.N., Piliposian, G.T.: Surface electro-elastic Love waves in a layered structure with a piezoelectric substrate and a dielectric layer. Int. J. Solids Struct. **44**, 5829–5847 (2007)
8. Danoyan, Z.N., Piliposian, G.T.: Surface electro-elastic shear horizontal waves in a layered structure with a piezoelectric substrate and a hard dielectric layer. Int. J. Solids Struct. **45**, 431–441 (2008)
9. Piliposian, G.T., Danoyan, Z.N.: Surface electro-elastic Love waves in a layered structure with a piezoelectric substrate and two isotropic layers. Int. J. Solids Struct. **46**, 1345–1353 (2009)

10. Liu, H., Wang, Z.K., Wang, T.J.: Effect of initial stresses on the propagation behaviour of Love waves in a layered piezoelectric structure. Int. J. Solid. Struct. **38**, 37–51 (2001)
11. Liu, H., Kuang, Z.B., Cai, Z.M.: Propagation of Bleustein-Gulyaev waves in a prestressed layered piezoelectric structure. Ultrasonics **41**, 397–405 (2003)
12. Jin, F., Wang, Z.K., Wang, T.J.: The propagation behaviour of Love waves in a pre-stressed piezoelectric layered structure. Key Eng. Mater. **183**(187), 755–760 (2000)
13. Jin, F., Wang, Z.K., Kishimoto, K.: The propagation behavior of B-G waves in a pre-stressed piezoelectric layered structure. Int. J. Nonlinear Sci. **4**, 125–138 (2003)
14. Qian, Z.H., Jin, F., Wang, Z.K., Kishimoto, K.: Love waves propagation of in a piezoelectric layered structures with initial stresses. Acta Mech. **171**, 41–57 (2004)
15. Jin, F., Qian, Z.H., Wang, Z.K., Kishimoto, K.: Propagation of Love waves in a piezoelectric layered structure with inhomogeneous initial stresses. Smart. Mater. Struct. **14**, 515–523 (2005)
16. Du, J.K., Jin, X., Wang, J.: Love wave propagation in layered magneto-electro-elastic structures with initial stress. Acta Mech. **192**, 169–189 (2007)
17. Li, X.Y., Wang, Z.K., Huang, S.H.: Love waves in functionally graded piezoelectric materials. Int. J. Solids Struct. **41**, 7309–7328 (2004)
18. Liu, J., Wang, Z.K.: The propagation behavior of Love waves in a functionally graded layered piezoelectric structure. Smart Mater. Struct. **14**, 137–146 (2005)
19. Du, J.K., Jin, X., Wang, J., Xian, K.: Love wave propagation in functionally graded piezoelectric material layer. Ultrasonics **46**, 13–22 (2007)
20. Qian, Z.H., Jin, F., Wang, Z.K., Kishimoto, K.: Transverse surface waves on a piezoelectric material carrying a functionally graded layer of finite thickness. Int. J. Eng. Sci. **45**, 455–466 (2007)
21. Qian, Z.H., Jin, F., Hirose, S.: Piezoelectric Love waves in an FGPM layered structure. Invited contribution to Special Issue on "Modeling of Functionally Graded Material Beams, Plates and Shells" (Guest Editors: E. Carrera and S. Brischetto) Mech. Adv. Mater. Struct. **18**, 77–84 (2011)
22. Liu, J., Cao, X.S., Wang, Z.K.: Propagation of Love waves in a smart functionally graded piezoelectric composite structure. Smart Mater. Struct. **16**, 13–24 (2007)
23. Jakoby, B., Vellekoop, M.J.: Properties of Love waves: applications in sensors. Smart Mater. Struct. **6**, 668–679 (1997)
24. Karl, F.G.: Wave Motion in Elastic Solids. Clarendon Press, Oxford (1975)