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Thermoacoustic response of a simply supported isotropic rectangular plate in graded thermal environments

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ABSTRACT

A theoretical model is developed to investigate the thermoacoustic response of a simply supported plate subjected to combined thermal and acoustic excitations, with two typical graded thermal environments considered. The thermoacoustic governing equation derived by incorporating the thermal moments, membrane forces and acoustic loads into the plate vibration equation is solved using the modal decomposition approach. In combination with the thermal boundary conditions, the Fourier heat conduction equation is solved for the graded temperature distribution in the plate. Fluid-structure coupling between acoustic excitation and the plate is ensured by adopting the velocity continuity condition at the fluid–plate interface. With focus placed on the effect of graded thermal environments on plate vibroacoustic response, numerical investigations reveal the necessity for considering thermal moments in theoretical modeling, particularly when graded thermal environments are of common concern for aircraft structures.

1. Introduction

The thermoacoustic response of thin structures in thermal environments has received increasing attention since the fuselages of supersonic/hypersonic vehicles are often exposed in severe aerodynamic, acoustic and thermal environments [1,2]. To protect devices in the vehicles, the thin fuselage structures should endure considerable graded thermal environment and acoustic excitation to make the devices work in normal temperature and low vibration environment. This work aims to develop a straightforward theoretical model to investigate the thermoacoustic response of a simply supported plate subjected to combined thermal and acoustic loads, which should be useful for the design of vehicle structures.

Great efforts have been devoted to investigating the mechanical and acoustical performance of elastic structures in thermal environments. Early in 1935, Maulbetsch [3] calculated theoretically thermal stress distributions in a simply supported rectangular plate due to exterior heating. Tsien [4] subsequently developed a theoretical model to calculate thermal stresses in heated wings, in which the differential equation for a heated plate was equivalent to that of an unheated plate by properly modifying the plate thickness and the loads. Jadeja and Loo [5] theoretically investigated heat-induced vibration of a rectangular plate with one edge fixed and the other three simply supported: the results indicated the possibility of predicting early fatigue failure.

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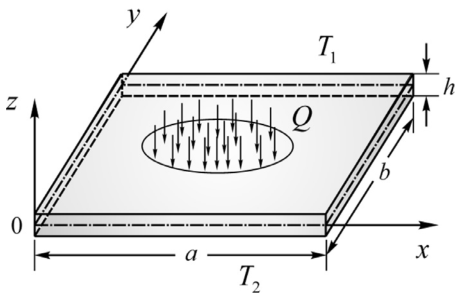

Fig. 1. Schematic illustration of a simply supported rectangular plate in graded thermal environment.

Concentrating on structure-borne noise transmission, Lyrintzis and Bofilios [6] presented an analytical model to account for the effects of elevated temperature, absorbed moisture and random external excitation on dynamic responses of stiffened composite plates and concluded that thermal and moisture effects are important for predicting such dynamic responses. With focus place upon mechanical deformation, Vel and Batra [2] proposed an exact solution for the deformation of a simply supported functionally graded rectangular plate under thermal loads. As an extension, by employing the third-order shear deformation plate theory to account for rotary inertia and transverse shear strains, Kim [7] formulated a theoretical model to investigate the vibration characteristics of an initially stressed functionally graded plate in thermal environments. Adopting the technique of coupled FEM (finite element method) and BEM (boundary element method), Jeyaraj et al. [8] studied the vibro-acoustic response of an isotropic plate in thermal environment. Considering further the inherent damping property of fiber-reinforced composite material, Jeyaraj et al. [9] investigated the vibro-acoustic response of a composite plate in thermal environment. Utilizing the commercial FEM code ABAQUS, Behnke et al. [10] conducted thermal-acoustic analysis for a metallic sandwich structure, which is constrained to have reduced-size so as to save computational efforts.

A comprehensive survey of literature reveals that there exists no study on sound transmission across a simply supported plate in graded thermal environment despite its practical significance for the design of aerodynamic heating structures. To address this deficiency, we develop a theoretical model to investigate the sound transmission response of a simply supported plate in two typical graded thermal environments, with thermal moments and membrane forces considered to account for the effect of thermal environment. The accuracy of theoretical model predictions is validated against numerical simulation results. The model is then used to investigate systematically the influence of graded thermal environment on the natural frequency and sound transmission performance of the plate.

2. Theoretical formulation

With reference to Fig. 1, consider a simply supported rectangular plate having length a, width b and thickness h, subjected to a combined thermal and acoustical excitation on its upper surface. To simulate the graded temperature environment of aircraft fuselage from interior to exterior, the lower surface (interior side) of the plate maintains a constant temperature T_2, while its upper surface (exterior side) is heated by constant temperature T_1 (or heat flux Q).

Under graded temperature environment, the simply supported plate of Fig. 1 endures in-plane forces and thermal moment, which can be described by the following vibration governing equation:

$$D\nabla^4 w + \rho h\frac{\partial^2 w}{\partial t^2} + \frac{1}{1-\nu}\nabla^2 M_T - j\omega(\rho_1\Phi_1 - \rho_2\Phi_2) = N_{xx}\frac{\partial^2 w}{\partial x^2} + N_{yy}\frac{\partial^2 w}{\partial y^2} + 2N_{xy}\frac{\partial^2 w}{\partial x\partial y}, \tag{1}$$

where w is the transverse displacement, D is the flexural rigidity, ρ is the density of plate material, ν is the Poisson ratio, ω is the angular frequency, (ρ_1, ρ_2) is the density of fluid media, and (Φ_1, Φ_2) is the velocity potential of sound wave in the upper and lower side of the plate. Subscripts 1 and 2 represent the exterior and interior side of the plate, respectively. M_T is the thermal moment defined by:

$$M_T = \alpha E\int_{-\frac{h}{2}}^{\frac{h}{2}} T(z)z dz, \tag{2}$$

and N_{xx}, N_{yy}, N_{xy} are the membrane forces given by:

$$N_{xx} = N_{yy} = -\frac{\alpha E}{1-\nu}\int_{-\frac{h}{2}}^{\frac{h}{2}} T(z)dz, \quad N_{xy} = 0. \tag{3}$$

In-plane stresses in the plate are expressed in terms of its displacement, as [11]:

$$\sigma_{xx} = -\frac{E}{1-\nu^2}\left[z\frac{\partial^2 w}{\partial x^2} + \nu z\frac{\partial^2 w}{\partial y^2} + (1+\nu)\alpha T(z)\right] \tag{4}$$

$$\sigma_{yy} = -\frac{E}{1-\nu^2}\left[z\frac{\partial^2 w}{\partial y^2} + \nu z\frac{\partial^2 w}{\partial x^2} + (1+\nu)\alpha T(z)\right] \tag{5}$$

$$\tau_{xy} = -\frac{Ez}{1+\nu}\frac{\partial^2 w}{\partial x \partial y}. \tag{6}$$

The incident time-harmonic plane sound pressure can be expressed in the form of velocity potential, as:

$$\Phi_1(x,y,z;t) = Ie^{-j\left(k_{1x}x+k_{1y}y-k_{1z}z-\omega t\right)} + \beta e^{-j\left(k_{1x}x+k_{1y}y+k_{1z}z-\omega t\right)}, \tag{7}$$

where I is the amplitude of the incident sound, β is the amplitude of the reflected sound, and $j=\sqrt{-1}$. The transmitted sound pressure in the form of velocity potential is:

$$\Phi_2(x,y,z;t) = \varepsilon e^{-j\left(k_{2x}x+k_{2y}y-k_{2z}z-\omega t\right)}, \tag{8}$$

where the wavenumber components in the x-, y- and z-directions (Fig. 1) are:

$$k_{1x} = k_1 \sin\varphi_1 \cos\theta, \quad k_{1y} = k_1 \sin\varphi_1 \sin\theta, \quad k_{1z} = k_1 \cos\varphi_1 \tag{9}$$

$$k_{2x} = k_2 \sin\varphi_2 \cos\theta, \quad k_{2y} = k_2 \sin\varphi_2 \sin\theta, \quad k_{2z} = k_2 \cos\varphi_2. \tag{10}$$

Here, φ_1 and φ_2 are the incident and transmitted elevation angle (from the outward normal direction of the plate plane), respectively; θ is the azimuth angle from the positive x-direction of the plate plane; and $k_1=\omega/c_1$ and $k_2=\omega/c_2$ are the wavenumber in the incident and transmitted side, respectively. The velocity and sound pressure in the acoustic field can be obtained by:

$$\hat{\mathbf{u}}_i = -\nabla\Phi_i, \quad p_i = \rho_i\frac{\partial\Phi_i}{\partial t} = j\omega\rho_i\Phi_i \quad (i=1,2,3). \tag{11}$$

As shown in Fig. 1, the plate is constrained by simply supported boundary conditions, requiring:

$$x=0,a: \quad w=0, \quad D\frac{\partial^2 w}{\partial x^2} + \frac{M_T}{1-\nu} = 0 \tag{12}$$

$$y=0,b: \quad w=0, \quad D\frac{\partial^2 w}{\partial y^2} + \frac{M_T}{1-\nu} = 0. \tag{13}$$

Consider next fluid–structure interaction. At the fluid–plate interface, the velocity of a fluid particle should equal that of the adjacent plate particle so that:

$$-\left.\frac{\partial\Phi_1}{\partial z}\right|_{z=h/2} = j\omega w, \quad -\left.\frac{\partial\Phi_2}{\partial z}\right|_{z=-h/2} = j\omega w. \tag{14}$$

Applying the modal decomposition approach, one can express the displacement of the plate as:

$$w(x,y,t) = \sum_{m,n} w_{mn}\varphi_{mn}(x,y)e^{j\omega t}, \tag{15}$$

where $\varphi_{mn}(x, y)$ is the modal function given by:

$$\varphi_{mn}(x,y) = \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}. \tag{16}$$

To facilitate the solution of the governing equations, the velocity potential for sound pressure waves may be written as:

$$\Phi_1(x,y,z;t) = \sum_{m,n} I_{mn}\varphi_{mn}e^{-j(-k_{1z}z-\omega t)} + \sum_{m,n}\beta_{mn}\varphi_{mn}e^{-j(k_{1z}z-\omega t)} \tag{17}$$

$$\Phi_2(x,y,z;t) = \sum_{m,n}\varepsilon_{mn}\varphi_{mn}e^{-j(-k_{2z}z-\omega t)}, \tag{18}$$

where I_{mn}, β_{mn} and ε_{mn} are the mnth amplitude of the velocity potential for incident, reflected and transmitted sound, respectively. These amplitudes are related to the general amplitudes I, β and ε in terms of Sine Fourier transform, as:

$$\lambda_{mn} = \frac{4}{ab}\int_0^b\int_0^a \lambda e^{-j\left(k_{ix}x+k_{iy}y\right)}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}dxdy(i=1,2), \tag{19}$$

from which the relationship between I_{mn} and I is obtained as:

$$I_{mn} = \frac{4mn\pi^2 I\left[1-(-1)^m e^{-jk_{1x}a} - (-1)^n e^{-jk_{1y}b} + (-1)^{m+n}e^{-j\left(k_{1x}a+k_{1y}b\right)}\right]}{\left(k_{1x}^2a^2 - m^2\pi^2\right)\left(k_{1y}^2b^2 - n^2\pi^2\right)}. \tag{20}$$

Substitution of Eqs. (16)–(18) into Eq. (14) leads to:

$$\beta_{mn} = I_{mn}e^{jk_{1z}h} + \frac{\omega}{k_{1z}}w_{mn}e^{jk_{1z}h/2} \tag{21}$$

$$\varepsilon_{mn} = -\frac{\omega w_{mn}}{k_{2z}}e^{ik_{2z}h/2}. \tag{22}$$

As previously mentioned, to simulate the graded temperature environment, one specific situation (Case 1) is the case where the interior side of the plate is maintained at constant temperature T_2 while its exterior side is heated by constant temperature T_1 (>T_2). Under such conditions, the temperature distribution in the plate along its thickness direction is governed by Fourier's law of heat conduction, as:

$$\rho C\frac{\partial T}{\partial t} = \kappa\frac{\partial^2 T}{\partial z^2}, \tag{23}$$

where C is the specific heat and κ is the thermal conductivity of the plate material. With the thermal boundary conditions specified as:

$$z = -\frac{h}{2}, \quad T = T_2 \tag{24}$$

$$z = \frac{h}{2}, \quad T = T_1 \tag{25}$$

Eq. (23) has a solution:

$$T = \frac{T_1 - T_2}{h}z + \frac{T_1 + T_2}{2}. \tag{26}$$

Thus the thermal moments and membrane forces in the plate are obtained as:

$$M_T = \alpha E\int_{-\frac{h}{2}}^{\frac{h}{2}} Tzdz = \frac{(T_1 - T_2)\alpha E h^2}{12} \tag{27}$$

$$N_{xx} = N_{yy} = -\frac{\alpha E}{1-\nu}\int_{-\frac{h}{2}}^{\frac{h}{2}} Tdz = -\frac{\alpha E(T_1 + T_2)h}{2(1-\nu)}. \tag{28}$$

For the alternative thermal situation of Case 2, the interior side of the plate maintains a constant temperature T_2 while its exterior side endures a fixed heat flux Q. In this case, the temperature distribution in the plate is governed by:

$$Q = -\kappa\frac{\partial T}{\partial z}. \tag{29}$$

Together with the following thermal boundary condition:

$$z = -\frac{h}{2}, \quad T = T_2. \tag{30}$$

Eq. (29) has a solution:

$$T = T_2 - \frac{Q}{\kappa}\left(z + \frac{h}{2}\right). \tag{31}$$

Correspondingly, the thermal moment and membrane forces are:

$$M_T = \alpha E\int_{-\frac{h}{2}}^{\frac{h}{2}} Tzdz = -\frac{\alpha E Q h^3}{12\kappa} \tag{32}$$

$$N_{xx} = N_{yy} = -\frac{\alpha E}{1-\nu}\int_{-\frac{h}{2}}^{\frac{h}{2}} Tdz = -\frac{\alpha E h}{(1-\nu)}\left(T_2 - \frac{Qh}{2\kappa}\right). \tag{33}$$

To favorably solve the governing equation, the thermal moment is expressed in terms of modal function, as:

$$M_T(x, y) = \sum_{m,n} a_{mn}\varphi_{mn}(x, y), \tag{34}$$

in which the amplitude a_{mn} is:

$$a_{mn} = \frac{4}{ab}\int_0^a\int_0^b M_T\varphi_{mn}(x, y)dxdy = 4M_T\frac{\left[1 - (-1)^m - (-1)^n + (-1)^{m+n}\right]}{mn\pi^2}. \tag{35}$$

Given the mnth modal vibration displacement and thermal moment as follows:

$$w(x, y, t) = \varphi_{mn}(x, y)e^{j\omega_{mn}t} \tag{36}$$

$$M_T(x,y) = a_{mn}\varphi_{mn}(x,y), \tag{37}$$

the free vibration of the plate is governed by:

$$D\nabla^4\varphi_{mn}(x,y) - \rho h\omega_{mn}^2\varphi_{mn}(x,y) + \frac{1}{1-\nu}a_{mn}\nabla^2\varphi_{mn}(x,y) = N_{xx}\nabla^2\varphi_{mn}(x,y). \tag{38}$$

In the presence of thermal moment and membrane forces, the mnth natural frequency of the plate can be derived from Eq. (38) as:

$$\omega_{mn} = \sqrt{\frac{D}{\rho h}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^2 - \frac{a_{mn}}{\rho h(1-\nu)}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right] + \frac{N_{xx}}{\rho h}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]}. \tag{39}$$

Following the weighted residual (Galerkin) method, by setting the integral of a weighted residual of the modal function to zero, an arbitrarily accurate series solution can be achieved. Multiplying the governing equation of (38) by the modal function and then integrating over the area of the plate, one has:

$$\int\int_A \begin{bmatrix} D\nabla^4 w + \rho h\dfrac{\partial^2 w}{\partial t^2} + \dfrac{1}{1-\nu}\nabla^2 M_T - j\omega(\rho_1\Phi_1 - \rho_2\Phi_2) \\ -\left(N_{xx}\dfrac{\partial^2 w}{\partial x^2} + N_{yy}\dfrac{\partial^2 w}{\partial y^2} + 2N_{xy}\dfrac{\partial^2 w}{\partial x\partial y}\right) \end{bmatrix} \cdot \varphi_{mn}(x,y)dA = 0. \tag{40}$$

The solution of this equation can be written as:

$$w_{mn} = \frac{\frac{1}{1-\nu}\left[(m\pi/a)^2 + (n\pi/b)^2\right]a_{mn} + 2j\omega\rho_1 I_{mn}e^{jk_{1z}h/2}}{D\left[(m\pi/a)^2 + (n\pi/b)^2\right]^2 - \rho h\omega^2 - j\omega^2(\rho_1/k_{1z} + \rho_2/k_{2z}) + N_{xx}\left[(m\pi/a)^2 + (n\pi/b)^2\right]}. \tag{41}$$

The sound power of the incident and transmitted sound is defined by:

$$\prod_i = \frac{1}{2}\mathrm{Re}\int\int_A p_i\cdot v_i^* dA, \tag{42}$$

where p_i is the sound pressure, $v_i = p_i/\rho_i c_i$ is the local fluid particle velocity, $\rho_i c_i$ is the characteristic impedance of the fluid, the subscript i= 1,2 represents the incident and transmitted sound, respectively, and the superscript asterisk denotes the complex conjugate. The transmission coefficient is defined as the ratio between transmitted sound power to incident sound power, as:

$$\tau(\varphi,\theta,T) = \frac{\Pi_2}{\Pi_1}. \tag{43}$$

Finally, the transmission loss of sound across the plate is given by:

$$STL = 10\log_{10}\left(\frac{1}{\tau}\right). \tag{44}$$

3. Numerical results and discussion

To determine the thermoacoustic responses of a simply supported plate under graded temperature environment, numerical calculation is performed under two idealized thermal conditions: the interior surface of the plate is maintained at constant temperature T_2 while its exterior surface is heated by constant temperature T_1 (Case 1) or constant heat flux Q (Case 2). To complete the numerical calculation, relevant physical parameters and structural dimensions are listed in Table 1. Of specific concern, the influence of graded temperature distribution on the natural frequency and transmission loss of the plate, as well as the effects of plate aspect ratio and sound incident angle on transmission loss are analyzed and discussed.

3.1. Computational validation

To access the accuracy and feasibility of the proposed theoretical model, we developed a coupled FEM-BEM (finite element method - boundary element method) numerical model to calculate the natural frequencies and sound transmission loss of the plate. The FEM numerical model is first developed with the software of ANSYS to calculate the natural frequencies of the plate. The resulting ANSYS file '.rst' is then loaded into the software of LMS Virtual.Lab to establish the coupled FEM-BEM model to calculate the sound transmission loss of the plate (as shown in Fig. 2). In the numerical model, all the geometrical and physical parameters of the plate and the air are kept the same as those used in the theoretical model. The boundary of the plate is set as simply supported. The length of each element is less than one-sixth of the acoustic wavelength at the highest frequency of interest to ensure the accuracy of the numerical model.

Table 1.
Structural dimensions and material properties.

Description	Parameter & Value
Elastic plate	
Length	$a = 1.2$ m
Width	$b = 0.8$ m
Thickness	$h = 0.05$ m
Young's modulus	$E = 70$ GPa
Density	$\rho = 2700$ kg/m^3
Poisson ratio	$\nu = 0.33$
Specific heat	$C= 902$ J/(kg·K)
Thermal conductivity	$\kappa = 236$ W/(m·K)
Thermal expansion coefficient	$\alpha= 2.3 \times 10^{-5}$ m/(m·K)
Acoustic fluid	
Density	$\rho_0 = 1.21$ kg/m^3
Sound speed	$c_0 = 343$ m/s

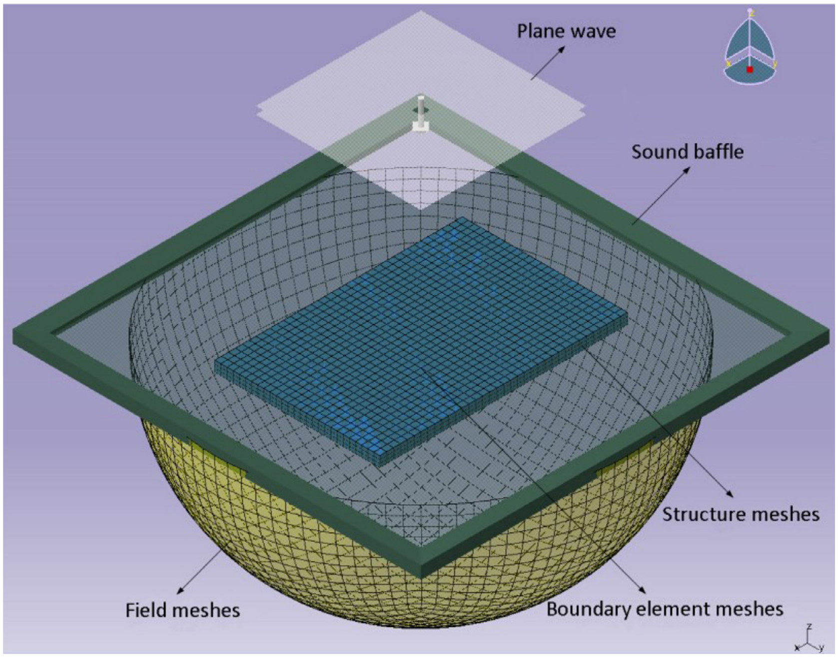

Fig. 2. The coupled FEM–BEM numerical model for sound transmission loss.

Given the incident acoustic wave with pressure p_0 and oblique (elevation) incident angle φ_1, the power of the incident acoustic wave can be calculated as:

$$W_i = \frac{p_0^2 \cos \varphi_1}{2\rho_0 c_0} S, \tag{45}$$

where ρ_0 and c_0 are the air density and the sound speed in air, and S is the area of the plate. The transmitted sound power W_t can be calculated via the coupled FEM-BEM numerical model in LMS Virtual.Lab. Applying the post-processing module of 'Function creator' and Eqs. (43) and (44), one can obtain the numerical results of transmission loss.

To validate the proposed theoretical model, Figs. 3 and 4 compare the theoretical predictions with numerical calculations for the natural frequencies and sound transmission loss of the simply supported isotropic rectangular plate, with T_1=T_2=100 °C. In the considered frequency range, the theoretical results match well with numerical results not only for the natural frequencies but also for the overall tendency of the transmission loss curve. The slightly larger value of the theoretical natural frequencies may be attributed to the ideal boundary handling of the theoretical model. To a large extent, these comparisons prove the feasibility and accuracy of the theoretical model, which can then be applied for the analyses presented in the following sections.

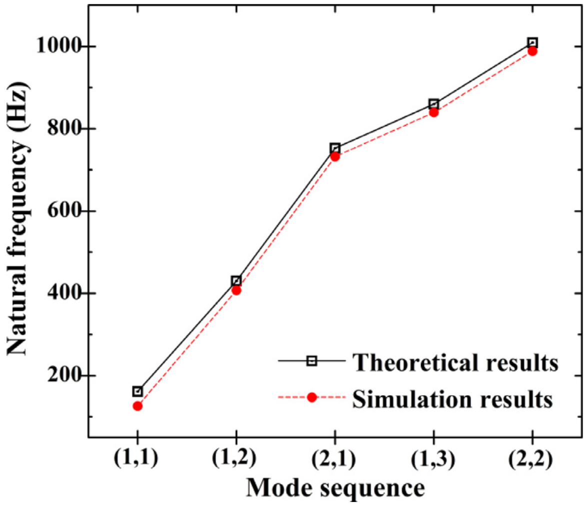

Fig. 3. Comparison of natural frequency between the theoretical model and the FEM numerical model.

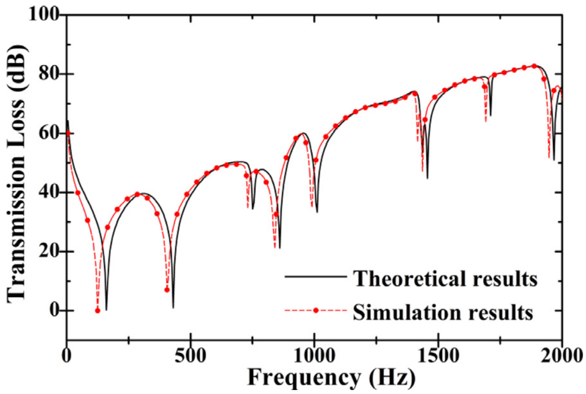

Fig. 4. Comparison of sound transmission loss between the theoretical model and the coupled FEM–BEM numerical model.

3.2. Natural frequency

To highlight the effect of graded temperature distribution, the natural frequencies of the first five order modes of the plate in uniform temperature environment are presented in Fig. 5 as reference. For comparison, the corresponding natural frequencies of the plate in graded temperature environment are presented in Figs. 6 and 7 for Case 1 and Case 2, respectively.

As shown in Fig. 5, the plate natural frequencies decrease as the uniform temperature is increased, which is attributed to the softening effect of the plate since its thermal moment and membrane forces increase with increasing temperature. As the plate is considered to be uniform and isotropic, the variation trend of its natural frequencies in negative gradient temperature environment is identical to that in positive gradient temperature environment. Further, because the elevation of the average plate temperature in Fig. 6 is smaller than that in Fig. 5, the corresponding increase of the natural frequencies in Fig. 6 is less significant.

The effect of graded temperature distribution induced by constant heat flux (Case 2) on plate natural frequency is presented in Fig. 7. As observed in Fig. 7(a), the natural frequencies almost remain unchanged when the heat flux is varied while is fixed. This can be explained by examining Eq. (31), in which the second term $Q(z+h/2)/\kappa$ is negligible relative to the first term T_2. Therefore, alteration of the heat flux plays an insignificant role in plate natural frequency. In contrast, all the natural frequencies noticeably decrease when T_2 is increased while the heat flux is fixed; Fig. 7(b).

3.3. Temperature distribution effect

Consider next the effect of uniform, negative and positive temperature distributions as well as heat flux environment on the transmission loss of a simply supported plate. As seen in Fig. 8 for the uniform temperature case, the transmission loss curve changes significantly when the plate temperature is altered. Remarkably, peaks and dips in the curve shift to

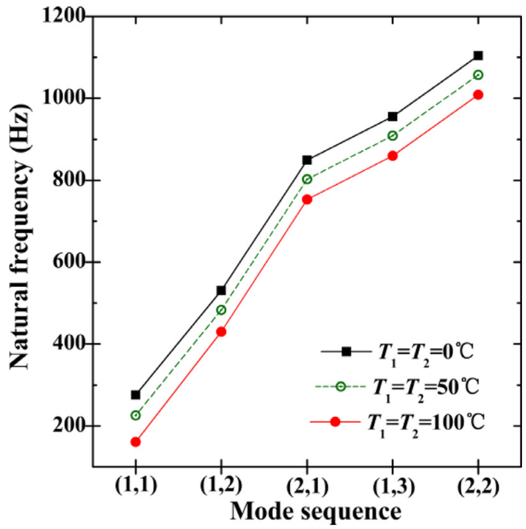

Fig. 5. Natural frequencies of a simply supported plate under uniform temperature environment.

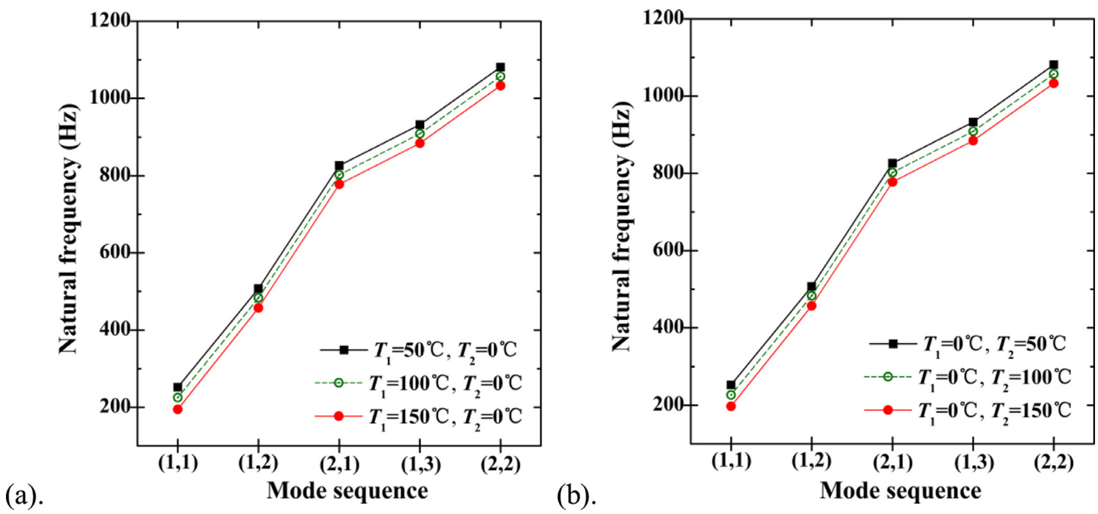

Fig. 6. Natural frequencies of a simply supported plate under Case 1 graded temperature environment: (a) negative gradient temperature; (b) positive gradient temperature.

lower frequencies as the temperature is increased, corresponding to decreased natural frequencies of plate anti-resonances and resonances. As aforementioned, the decreasing natural frequency is attributed to the softening effect of the plate in the presence of thermal membrane forces.

In comparison to the uniform temperature case of Fig. 8, it is seen from Figs. 9 and 10 that the transmission loss of the plate for Case 1 graded temperature environment decreases over a wide frequency range as the temperature gradient is increased, apart from the shift of the peaks and dips to lower frequencies. This is due to the appearance of thermal moments in graded temperature environment, which do not exist in the uniform temperature case. Actually, this signifies the importance of thermal moments in the vibroacoustic behavior of the plate, even if the temperature gradient in the plate is relatively small. Again, as a result of the uniform and isotropic nature of the plate material considered here, the plate in negative gradient temperature environment has identical vibro-acoustic performance as that in positive gradient environment.

As indicated in Figs. 11 and 12, the variation trend of the transmission loss curve changes significantly under Case 2 graded temperature environment. It is seen from Fig. 11 that the locations of the peaks and dips remain almost unchanged, consistent with the results of Fig. 7(a). This is because the change in heat flux Q does not noticeably alter the plate temperature distribution, since the term $Q(z+h/2)/\kappa$ is negligible relative to T_2 in Eq. (31). However, the transmission loss remarkably decreases over the whole frequency range, except for the resonance dips, if the heat flux is increased while holding the temperature T_2 unchanged. Under such conditions, although the plate temperature distribution does not noticeably change,

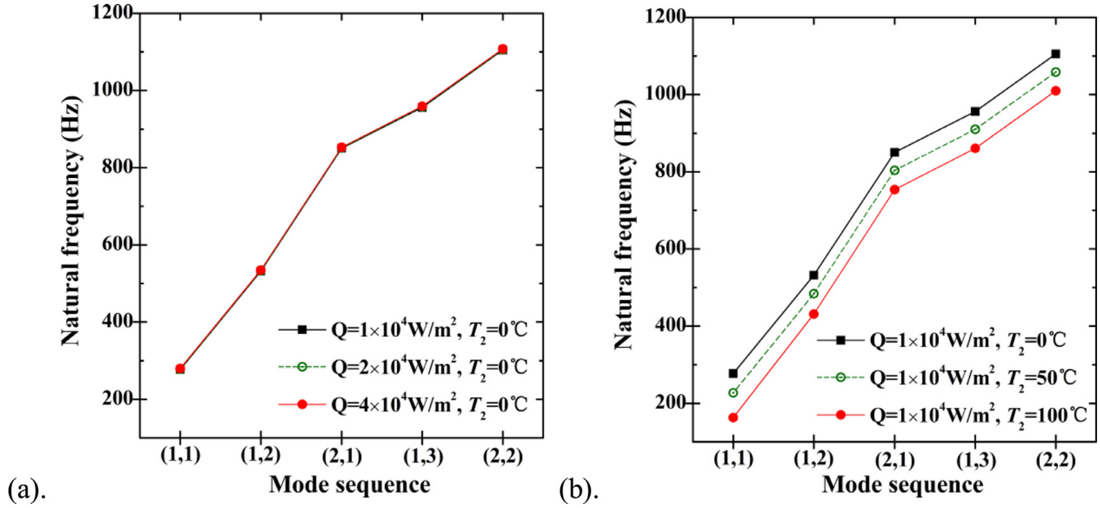

Fig. 7. Natural frequencies of a simply supported plate under Case 1 graded temperature environment: (a) varying heat flux on exterior surface of plate; (b) varying temperature on interior surface of plate.

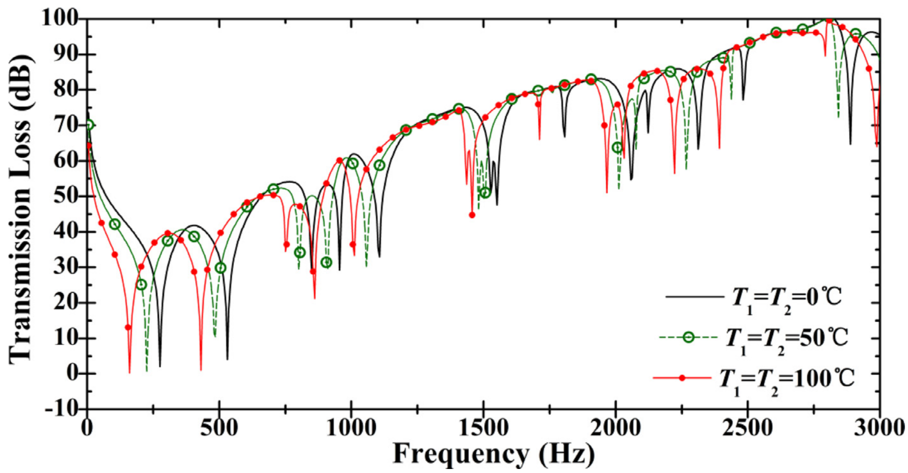

Fig. 8. Transmission loss of a simply supported plate under uniform temperature environment.

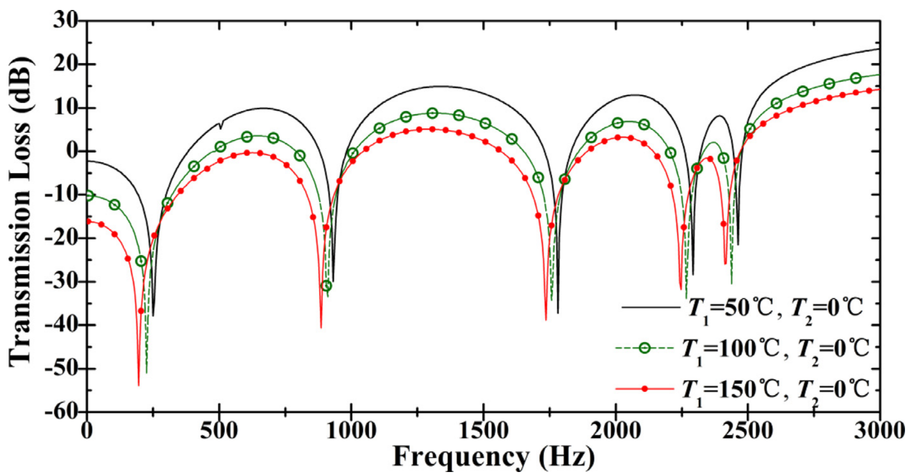

Fig. 9. Transmission loss of a simply supported plate under negative gradient temperature environment (Case 1).

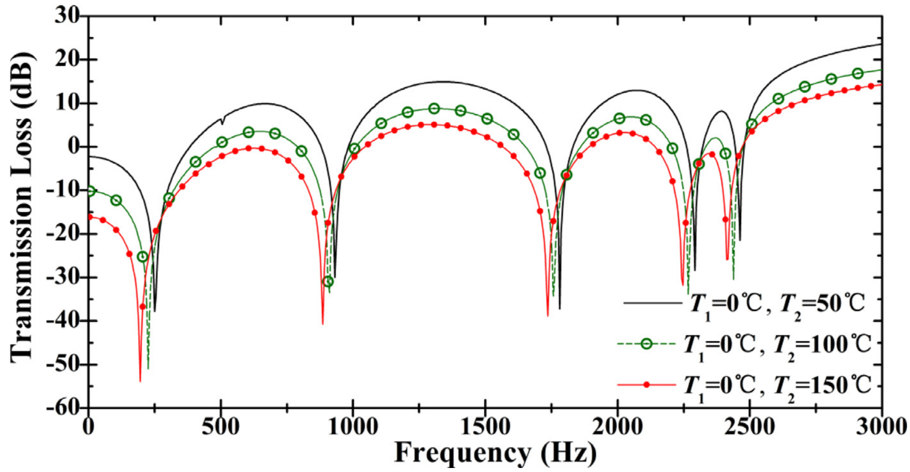

Fig. 10. Transmission loss of a simply supported plate under positive gradient temperature environment (Case 1).

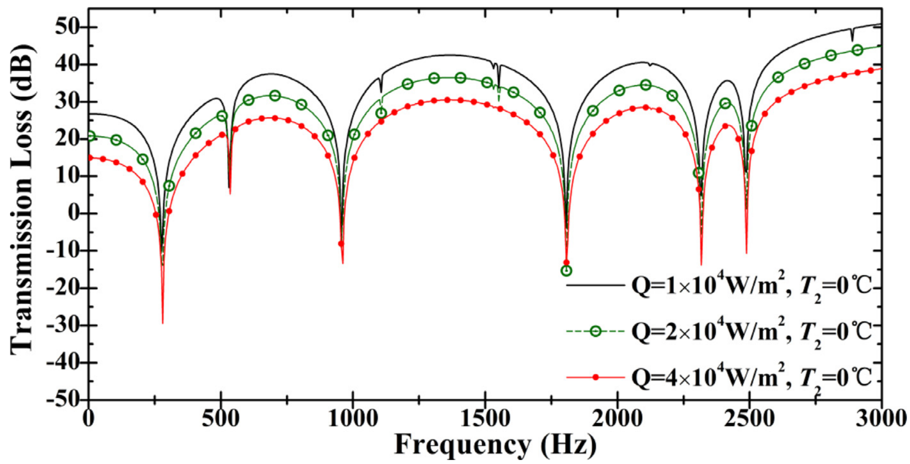

Fig. 11. Transmission loss of a simply supported plate under heat flux environment (Case 2): influence of heat flux.

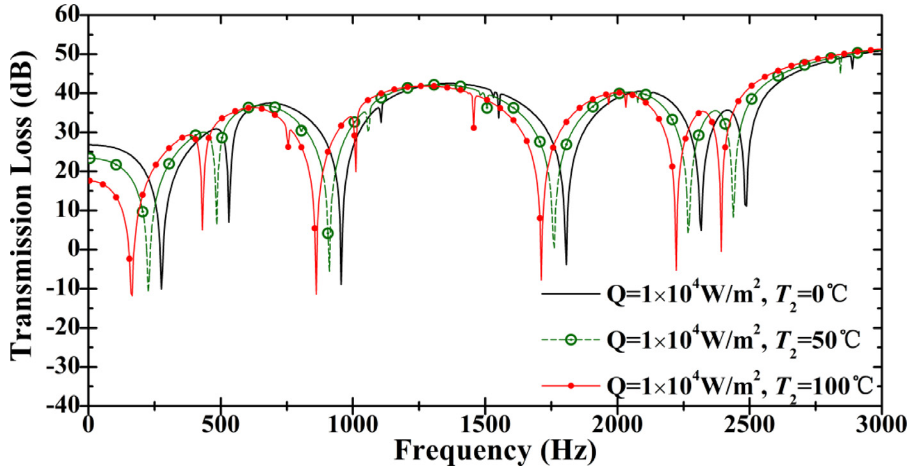

Fig. 12. Transmission loss of a simply supported plate under heat flux environment (Case 2): influence of temperature gradient.

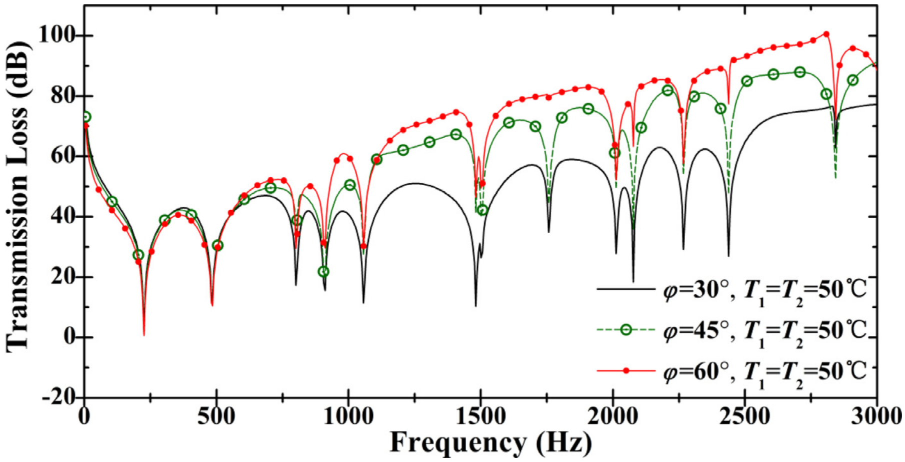

Fig. 13. Transmission loss of a simply supported plate under uniform temperature environment: effect of elevation incident angle.

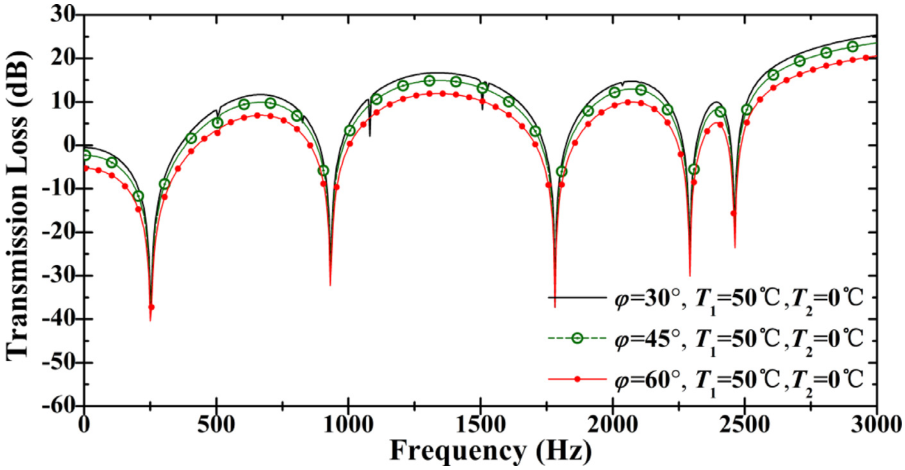

Fig. 14. Transmission loss of a simply supported plate under Case 1 graded temperature environment: effect of elevation incident angle.

the increased thermal moments induced by increasing heat flux affect significantly the tendency of the transmission loss curve.

In contrast to Fig. 11, when temperature T_2 is increased and heat flux Q is fixed, the peaks and dips in Fig. 12 all move to lower frequencies and the transmission loss does not change in value significantly. Increasing T_2 dramatically changes the temperature distribution in the plate, altering therefore its natural frequencies. However, the thermal moments do not change since the heat flux is fixed, and hence there is insignificant variation of the transmission loss value in Fig. 12.

3.4. Sound incident angle effect

In the absence of temperature gradient, it is known that the transmission loss of a simply supported plate varies with plane sound incident angle. In contrast, as non-uniform temperature distribution in the plate induces thermal moments and membrane forces, it should also affect the influence of sound incident angle on transmission loss, as evidenced by the results shown in Figs. 13, 14 and 15 for uniform temperature as well as Case 1 and Case 2 graded temperature environments.

In general, at room temperature, the transmission loss decreases with increasing elevation incident angle [12,13]. In contrast, when the plate is in uniform temperature environment, it is observed from Fig. 13 that the transmission loss significantly increases as the elevation angle is increased in the high frequency range but remains unchanged in the low frequency range. This is because the membrane force caused by elevated temperature stiffens the structure and thus blocks the propagation of flexural waves in the plate. Therefore, an incident sound with a larger elevation angle is more difficult to penetrate across the plate. However, when thermal moment is induced by graded temperature distribution through the plate thickness, the above tendency is completely changed. As shown in Figs. 14 and 15, the transmission loss decreases as the elevation angle is increased. This phenomenon confirms that the effect of thermal moment on transmission loss overwhelms that of thermal membrane forces.

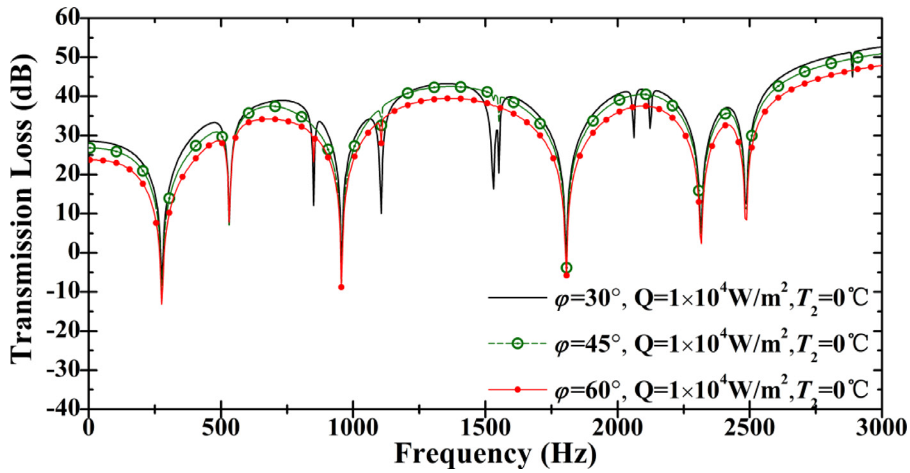

Fig. 15. Transmission loss of a simply supported plate under Case 2 graded temperature environment: effect of elevation incident angle.

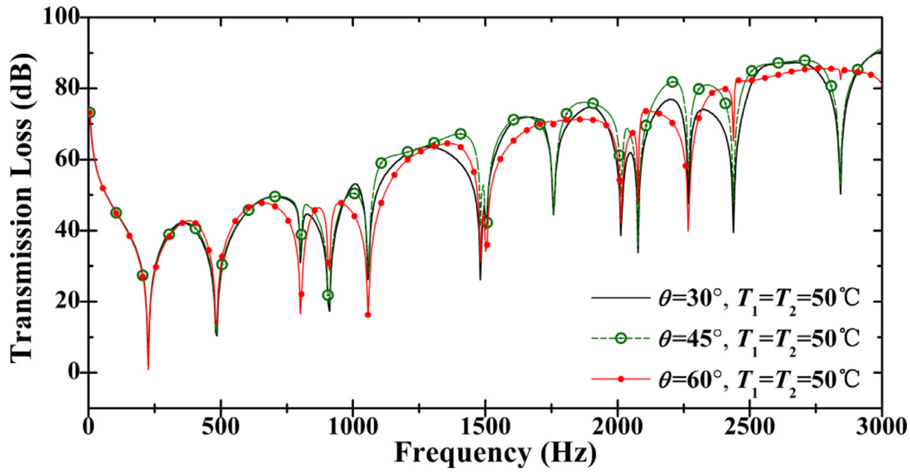

Fig. 16. Transmission loss of a simply supported plate under uniform temperature environment: effect of azimuth incident angle.

The effect of sound incident azimuth angle on the transmission performance of the plate is evaluated in different thermal environments, as shown in Figs. 16, 17 and 15, with the incident elevation angle fixed at 45° There exist visible discrepancies between the transmission loss curves for different azimuth angles when the plate is placed in uniform elevated temperature environment (Fig. 16). In comparison, when the plate is in graded temperature environment, the discrepancy among the transmission loss curves for different azimuth angles tends to be invisible, as shown in Figs. 17 and 18. Consequently, accounting for thermal moments in theoretical modeling is important, especially when temperature gradient in the simply supported plate is relatively large.

4. Conclusions

A theoretical model for the thermoacoustic response of a simply supported isotropic rectangular plate placed in graded thermal environment is developed. The thermoacoustic governing equation of the fluid–structure system is formulated by incorporating the thermal moments, membrane forces and acoustical excitation into the plate vibration equation. The thermal moments and membrane forces arising from graded temperature distribution in the plate are derived by integrating the stresses through the plate thickness, which can be obtained once the temperature field is determined from Fourier's law of heat conduction. The acoustical excitation is coupled with plate vibration via fluid–structure coupling. By adopting the mode function for simply supported condition, the thermoacoustic governing equation is solved by using the modal decomposition approach. Ultimately, the sound transmission loss of the plate is expressed in the form of the ratio between incident sound power and transmitted sound power in decibel scale. Numerical simulations are carried out to validate the proposed theoretical model, with good agreement achieved.

The theoretical model is employed to quantify the effect of graded temperature environment on the natural frequencies and vibroacoustic performance of the simply supported plate. The natural frequencies of the plate increase with elevated

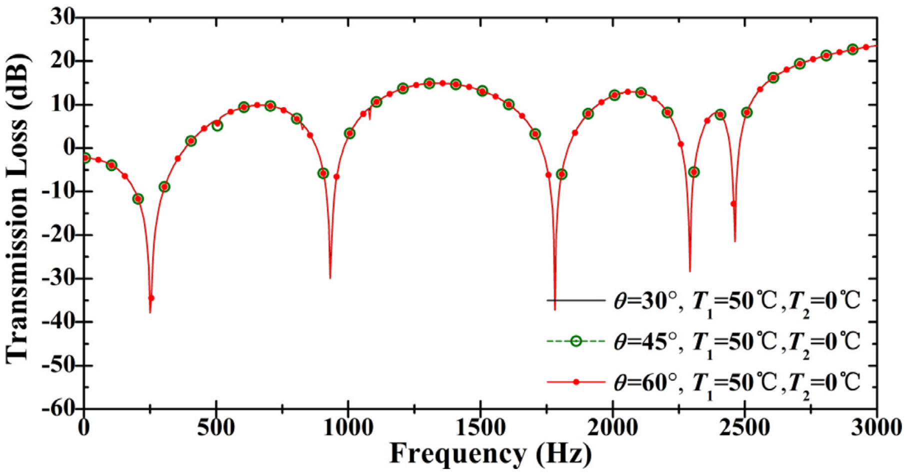

Fig. 17. Transmission loss of a simply supported plate under Case 1 graded temperature environment: effect of azimuth incident angle.

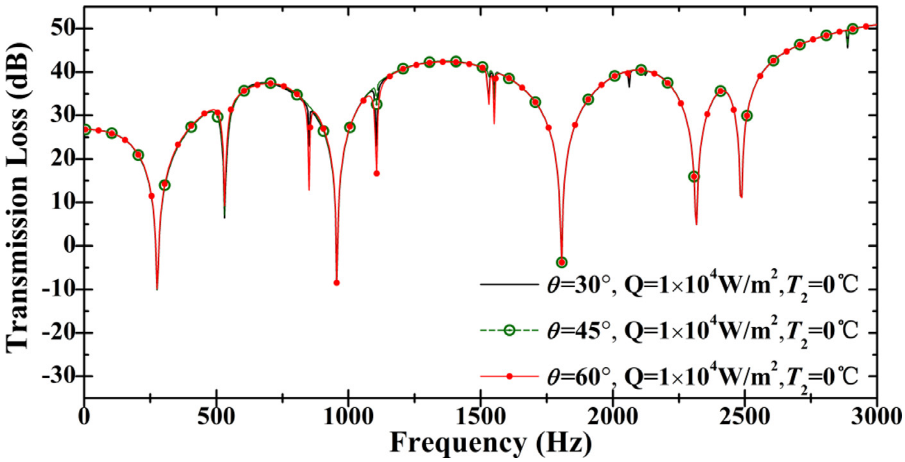

Fig. 18. Transmission loss of a simply supported plate under Case 2 graded temperature environment: effect of azimuth incident angle.

temperature of the plate, while heat flux plays an insignificant role. Relative to uniform temperature environment, the transmission loss of the plate in graded temperature environment decreases significantly over a wide frequency range as the temperature gradient is increased, due to increasing thermal moments in the plate. It is of vital importance to account for such thermal moments in theoretical analysis since graded temperature environments are commonly found in aircraft fuselage structures. In uniform elevated temperature environments, the transmission loss significantly increases with increasing elevation angle in high frequency range, due mainly to thermal membrane forces in the plate. In graded temperature environments, the transmission loss decreases with increasing elevation angle, due mainly to the appearance of plate thermal moments. The effect of sound incident azimuth angle on transmission loss can be neglected when the plate is placed in graded thermal environments.

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