Philosophical Magazine Letters

ISSN: 0950-0839 (Print) 1362-3036 (Online) Journal homepage: http://www.tandfonline.com/loi/tphl20

Shear-horizontal surface waves in a half-space of piezoelectric semiconductors

Chunlong Gu & Feng Jin

To cite this article: Chunlong Gu & Feng Jin (2015) Shear-horizontal surface waves in a half-space of piezoelectric semiconductors, Philosophical Magazine Letters, 95:2, 92-100, DOI: 10.1080/09500839.2015.1011249

To link to this article: http://dx.doi.org/10.1080/09500839.2015.1011249

Published online: 02 Apr 2015.

Philosophical Magazine Letters, 2015
Vol. 95, No. 2, 92–100, http://dx.doi.org/10.1080/09500839.2015.1011249

Shear-horizontal surface waves in a half-space of piezoelectric semiconductors

Chunlong Gu and Feng Jin*

State Key Laboratory for Strength and Vibration of Mechanical Structures, Xi'an Jiaotong University, Xi'an, Shaanxi 710049, China

(*Received 18 October 2014; accepted 18 January 2015*)

We study the propagation of shear-horizontal waves near the surface of a piezoelectric semiconductor half-space of crystals of class 6 mm with the presence of a biasing electric field in the propagation direction. The three-dimensional equations of linear piezoelectric semiconductors are used. A transcendental equation that determines the dispersion relation is obtained and solved numerically. Results show that the semiconduction affects the wave speed and causes wave dispersion as well as attenuation, and that the waves can be amplified by the biasing electric field.

Keywords: shear-horizontal surface waves; piezoelectric semiconductors; dispersion relation; propagation and amplification of the Bleustein–Gulyaev waves

1. Introduction

Piezoelectric materials have been widely used to make electromechanical devices. They are usually treated as dielectric although some of them are in fact semiconductors [1]. An acoustic wave propagating in a piezoelectric crystal is accompanied by an electric field. When the crystal is also semiconducting, the electric field produces currents and space charge resulting in dispersion and acoustic loss [2]. The interaction between a travelling acoustic wave and mobile charges in piezoelectric semiconductors is called the acoustoelectric effect [3]. Acoustoelectric effect can also be realized using composite materials or structures of piezoelectric dielectrics and non-piezoelectric semiconductors [4,5]. In these composites, the acoustoelectric effect is due to the combination of the piezoelectric effect and semiconduction in each component phase.

Researchers have made various attempts to make use of the semiconduction in certain piezoelectric materials for devices. It was found that an acoustic wave travelling in a piezoelectric semiconductor can be amplified by the application of a dc electric field [6–9]. This phenomenon is called acoustoelectric amplification of acoustic waves. Through the electric field accompanying a propagating acoustic wave in a piezoelectric semiconductor, carriers can be transported by the acoustic wave from one place to another (acoustic charge transport) [10,11]. Relatively more recently, people have been trying to use the electric field produced by mechanical deformations in a piezoelectric

*Corresponding author. Email: jinfengzhao@263.net

semiconductor to make devices such as strain sensors (piezoelectronics) [12] and energy harvesters [13–16].

For acoustoelectric amplification and acoustic charge transport, people have studied various wave propagation problems in piezoelectric semiconductors, e.g. [17–19]. In a half-space of non-conducting piezoelectric ceramics or crystals of class 6 mm, it is well known that a shear-horizontal surface wave can propagate and is called the Bleustein–Gulyaev wave [20,21]. It was shown in [7] that in a composite structure of a non-conducting piezoelectric half-space with a non-piezoelectric semiconductor film, the Bleustein–Gulyaye wave can be amplified by a biasing electric field. In [7], the piezoelectric effect and the semiconduction exist separately in the half-space and the film. In this paper, we study the propagation and amplification of the Bleustein–Gulyaev wave in a true piezoelectric semiconducting half-space without the need of a surface film.

2. Governing equations

The basic behaviour of piezoelectric semiconductors can be described by a linear phenomenological theory [2,6,22]. Consider a one-carrier piezoelectric semiconductor. It is under a uniform biasing electric field $\bar{E}_j$. The carrier charge and steady-state carrier density are q and $\bar{n}$, respectively. When an acoustic wave propagates through the material, it produces an incremental electric field E_j and electric current J_i. The perturbation of the carrier density is denoted by n. The linear theory for the small and dynamic signals consists of the equations of motion (Newton's law), Gauss's law of electrostatics and the conservation of charge:

$$\begin{aligned} T_{ji,j} &= \rho \ddot{u}_i, \\ D_{i,i} &= qn, \\ q\dot{n} + J_{i,i} &= 0, \end{aligned} \tag{1}$$

where u_i is the displacement vector, T_{ij} is the stress tensor, ρ is the mass density and D_i is the electric displacement vector. The Cartesian tensor notation and summation convention for repeated indices are employed. A comma followed by an index denotes partial differentiation with respect to the coordinate associated with the index. A superimposed dot represents differentiation with respect to time t. The above equations are accompanied by the following constitutive relations:

$$\begin{aligned} T_{ij} &= c_{ijkl}S_{kl} - e_{kij}E_k, \\ D_i &= e_{ijk}S_{jk} + \varepsilon_{ij}E_j\ , \\ J_i &= q\bar{n}\mu_{ij}E_j + qn\mu_{ij}\bar{E}_j - qd_{ij}n_{,j}, \end{aligned} \tag{2}$$

where the strain tensor S_{ij} and the electric field E_k are related to the displacement u_i and the electric potential Φ by:

$$\begin{aligned} S_{ij} &= (u_{i,j} + u_{j,i})/2, \\ E_i &= -\Phi_{,i}. \end{aligned} \tag{3}$$

In (2), c_{ijkl}, e_{kij} and ε_{ij} are the elastic, piezoelectric and dielectric constants. μ_{ij} and d_{ij} are the carrier mobility and diffusion constants. With successive substitutions from (2) and (3), we can rewrite (1) as equations for u_i, Φ and n:

$$
\begin{aligned}
c_{ijkl}u_{k,lj} + e_{kij}\Phi_{,kj} &= \rho\ddot{u}_i, \\
e_{ikl}u_{k,li} - \varepsilon_{ij}\Phi_{,ij} &= qn, \\
\dot{n} - \bar{n}\mu_{ij}\Phi_{,ij} + \mu_{ij}\bar{E}_j n_{,i} - d_{ij}n_{,ij} &= 0.
\end{aligned} \tag{4}
$$

On the boundary of a finite body with a unit exterior normal n_i, the mechanical displacement u_i or the traction vector $T_{ij}n_i$, the electric potential Φ or the normal component of the electric displacement vector $D_i n_i$, the carrier density perturbation n or the normal current $J_i n_i$ or the combinations of some of them may be prescribed [23].

3. Antiplane problems

Bleustein–Gulyaev waves belong to antiplane or shear-horizontal motions. In this section, we specialize the general equations in the previous section to antiplane motions of crystals of class 6 mm. Consider the piezoelectric semiconductor half-space in Figure 1. Antiplane motions are described by the following fields:

$$
\begin{aligned}
u_3 = u_3(x_1, x_2, t), \quad u_1 = u_2 = 0, \\
\Phi = \Phi(x_1, x_2, t), \\
n = n(x_1, x_2, t).
\end{aligned} \tag{5}
$$

In this case, some of the strain, stress, electric field, electric displacement and current components vanish. For crystals of class 6 mm, the remaining stress, electric displacement and current components are:

$$
\begin{aligned}
T_{13} &= c_{44}u_{3,1} + e_{15}\Phi_{,1}, \\
T_{23} &= c_{44}u_{3,2} + e_{15}\Phi_{,2}, \\
D_1 &= e_{15}u_{3,1} - \varepsilon_{11}\Phi_{,1}, \\
D_2 &= e_{15}u_{3,2} - \varepsilon_{11}\Phi_{,2}, \\
J_1 &= q\bar{n}\mu_{11}E_1 - qd_{11}n_{,1}, \\
J_2 &= q\bar{n}\mu_{11}E_2 + qn\mu_{11}\bar{E}_2 - qd_{11}n_{,2},
\end{aligned} \tag{6}
$$

where $\bar{E}_1 = 0$ was assumed. $\bar{E}_2$ is in the wave propagation direction and is kept for the study of the acoustoelectric amplification of the waves. The substitution of (5) and (6) into $(1)_1$ (when the index $i = 3$) and $(1)_{2,3}$ gives:

$$
\begin{aligned}
c_{44}\nabla^2 u_3 + e_{15}\nabla^2\Phi &= \rho\ddot{u}_3, \\
e_{15}\nabla^2 u_3 - \varepsilon_{11}\nabla^2\Phi &= qn, \\
\dot{n} - \bar{n}\mu_{11}\nabla^2\Phi - d_{11}\nabla^2 n + n_{,2}\mu_{11}\bar{E}_2 &= 0,
\end{aligned} \tag{7}
$$

which are the equations needed for antiplane motions.

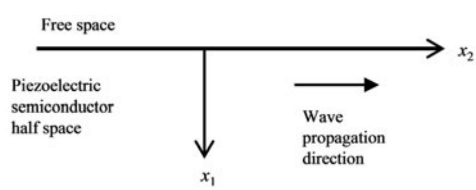

Figure 1. A piezoelectric semiconductor half-space and coordinate system.

4. Propagation of surface waves

For surface waves propagating in the x_2 direction over the half-space in Figure 1, we look for a solution in the following form:

$$\begin{aligned} u_3 &= Ae^{-k\beta x_1}e^{ik(x_2-ct)}, \\ \Phi &= Be^{-k\beta x_1}e^{ik(x_2-ct)}, \\ n &= De^{-k\beta x_1}e^{ik(x_2-ct)}, \end{aligned} \tag{8}$$

where c is the wave speed, k is the wave number which is taken to be real and positive and β describes the decay rate from the free surface at $x_1 = 0$ which should have a positive real part. A, B and D represent the wave amplitudes. Substituting (8) into (7), cancelling the common exponential factor, we obtain the following homogeneous linear equations for A, B and D:

$$\begin{aligned} \left[c_{44}\left(\beta^2-1\right)+\rho c^2\right]A + e_{15}\left(\beta^2-1\right)B &= 0, \\ e_{15}k^2\left(\beta^2-1\right)A - \varepsilon_{11}k^2\left(\beta^2-1\right)B - qD &= 0, \\ \bar{n}\mu_{11}k^2\left(\beta^2-1\right)B + \left[d_{11}k^2\left(\beta^2-1\right)+ikc-ik\mu_{11}\bar{E}_2\right] &= 0. \end{aligned} \tag{9}$$

For non-trivial solutions of A, B and D, the determinant of the coefficient matrix of (9) must vanish, which yields an algebraic equation for β^2. Denoting $\beta^2-1=\alpha$, we can write the equation for β^2 as a cubic equation for α**:**

$$\begin{aligned} \alpha\Big\{d_{11}k^2\left(c_{44}\varepsilon_{11}+e_{15}^2\right)\alpha^2 + \left[d_{11}\varepsilon_{11}\rho c^2k^2 + ik(c-\mu_{11}\bar{E}_2)\left(c_{44}\varepsilon_{11}+e_{15}^2\right) - c_{44}q\bar{n}\mu_{11}\right]\alpha \\ + \left[ik(c-\mu_{11}\bar{E}_2)\varepsilon_{11} - q\bar{n}\mu_{11}\right]\rho c^2\Big\} = 0. \end{aligned} \tag{10}$$

Since (10) has two factors, the three roots of (10) can be easily written as:

$$\begin{aligned} \alpha_1 &= 0, \\ \alpha_2 &= \frac{-b' + \sqrt{b'^2-4a'c'}}{2a'}, \\ \alpha_3 &= \frac{-b' - \sqrt{b'^2-4a'c'}}{2a'}, \end{aligned} \tag{11}$$

where

$$\begin{aligned} a' &= d_{11}k^2\left(c_{44}\varepsilon_{11}+e_{15}^2\right), \\ b' &= d_{11}\varepsilon_{11}\rho c^2k^2 + ik(c-\mu_{11}\bar{E}_2)\left(c_{44}\varepsilon_{11}+e_{15}^2\right) - c_{44}q\bar{n}\mu_{11}, \\ c' &= \left[ik(c-\mu_{11}\bar{E}_2)\varepsilon_{11} - q\bar{n}\mu_{11}\right]\rho c^2. \end{aligned} \tag{12}$$

α depends on the wave speed c and the wave number k. Once α is known, β can be obtained from $\alpha = \beta^2 - 1$. Only the three roots of β with positive real parts are taken and are denoted by β_m (m = 1, 2, 3). Corresponding to each β_m, the ratios among the amplitudes A, B and D can be determined from (9) as:

$$\begin{aligned} P_m &= \frac{A_m}{B_m} = -\frac{e_{15}\left(\beta_m^2-1\right)}{c_{44}\left(\beta_m^2-1\right)+\rho c^2}, \\ Q_m &= \frac{D_m}{B_m} = \frac{k^2}{q}\left(\beta_m^2-1\right)\left(e_{15}P_m - \varepsilon_{11}\right). \end{aligned} \tag{13}$$

Then the general surface wave solution in the form of (8) satisfying (7) and the decay condition at $x_1 = +\infty$ can be written as:

$$\begin{aligned} u_3 &= \sum_{m=1}^{3} P_m B_m e^{-k\beta_m x_1} e^{ik(x_2-ct)}, \\ \Phi &= \sum_{m=1}^{3} B_m e^{-k\beta_m x_1} e^{ik(x_2-ct)}, \\ n &= \sum_{m=1}^{3} Q_m B_m e^{-k\beta_m x_1} e^{ik(x_2-ct)}, \end{aligned} \tag{14}$$

where B_m ($m = 1, 2, 3$) are undetermined constants. With (14), T_{13}, D_1 and J_1 can be calculated from (6) which are needed in boundary and continuity conditions.

We consider a unelectroded half-space. In the free space above the half-space, the electric potential and the normal electric displacement component relevant to boundary/continuity conditions can be written as [20]:

$$\begin{aligned} \Phi_0 &= \sum_{m=1}^{3} B_m e^{kx_1} e^{ik(x_2-ct)}, \\ D_0 &= -\varepsilon_0 \Phi_{0,1} = -\varepsilon_0 k \sum_{m=1}^{3} B_m e^{kx_1} e^{ik(x_2-ct)}, \end{aligned} \tag{15}$$

which automatically satisfies the continuity of the electric potential, i.e. $\Phi = \Phi_0$ at the surface of the half-space.

At the surface of the half-space, the remaining boundary and continuity conditions are:

$$\begin{aligned} T_{13} &= 0, \\ \dot{D}_1 + J_1 &= \dot{D}_0, \\ n &= 0. \end{aligned} \tag{16}$$

$(16)_2$ represents the conservation of charge at the surface [23] which is more general than the corresponding one in [20] because of semiconduction. Compared to the continuity conditions in [20] for non-conducting piezoelectrics, $(16)_3$ is a new boundary condition because of the additional equation from semiconduction. Physically it means that the perturbation of the carrier density vanishes at the boundary. Substituting (14) and (15) into (16), we obtain a system of homogeneous linear equations for B_m:

$$\begin{cases} \sum_{m=1}^{3} (c_{44} P_m + e_{15}) \beta_m B_m = 0, \\ \sum_{m=1}^{3} [(e_{15} P_m - \varepsilon_{11}) i k^2 c \beta_m + (\bar{n}\mu_{11} + d_{11} Q_m) q k \beta_m - i k^2 \varepsilon_0 c] B_m = 0, \\ \sum_{m=1}^{3} Q_m B_m = 0. \end{cases} \tag{17}$$

The determinant of the coefficient matrix of (17) has to vanish, which gives an equation that determines the wave speed c vs. the wave number k. The equation is solved numerically. The wave speed thus determined is a complex number whose real and imaginary parts represent the real or true wave speed and wave attenuation, respectively.

5. Numerical results and discussion

For numerical results consider a half-space of ZnO with [24] $c_{44} = 43$ GPa, $e_{15} = -0.48$ C/m^2, $\varepsilon_{11} = 7.61 \times 10^{-11}$ F/m, $\rho = 5700$ kg/m^3, $q = 1.602 \times 10^{-19}$ C, $\mu_{11} = 1$ m^2/Vs and $d_{11} = \mu_{11}kT/q$ [25]. k is the Boltzmann constant and T is the absolute temperature. At room temperature, $kT/q_e = 0.026$ V [25] where $q_e = 1.602 \times 10^{-19}$ coulomb is the electronic charge. For the carriers, we consider holes with $q = q_e$. Present technology can make a material with $\bar{n}$ of any value between zero and 10^{19}/m^3 [26,27]. $\bar{n}$ will be varied in the following. We use the speed of the Bleustein–Gulyaev wave in a non-conducting piezoelectric half-space as a normalizing speed:

$$v_{BG}^2 = \frac{\bar{c}_{44}}{\rho}\left[1 - \frac{\bar{k}_{15}^4}{(1 + \varepsilon_{11}/\varepsilon_0)^2}\right], \bar{c}_{44} = c_{44} + \frac{e_{15}^2}{\varepsilon_{11}}, \bar{k}_{15}^2 = \frac{e_{15}^2}{\varepsilon_{11}\,\bar{c}_{44}}. \tag{18}$$

5.1. *Effects of semiconduction on wave speed and attenuation*

Consider the case without a biasing electric field first, i.e. $\bar{E}_2 = 0$. The basic effects of semiconduction on the complex wave speed are shown in Figures 2a and b for different values of the unperturbed carrier density $\bar{n}$.

The normalized real part of c or the true wave speed is shown in Figure 2a. Different from the Bleustein–Gulyaev wave which is non-dispersive, the wave is dispersive now. The wave speed is smaller than one, showing that the semiconduction lowers the wave speed. The effect is stronger for smaller values of k or long waves with low frequencies but is still relatively weak, only about a few per cent. For large k or short waves with high frequencies, the electric field reverses its direction quickly which is

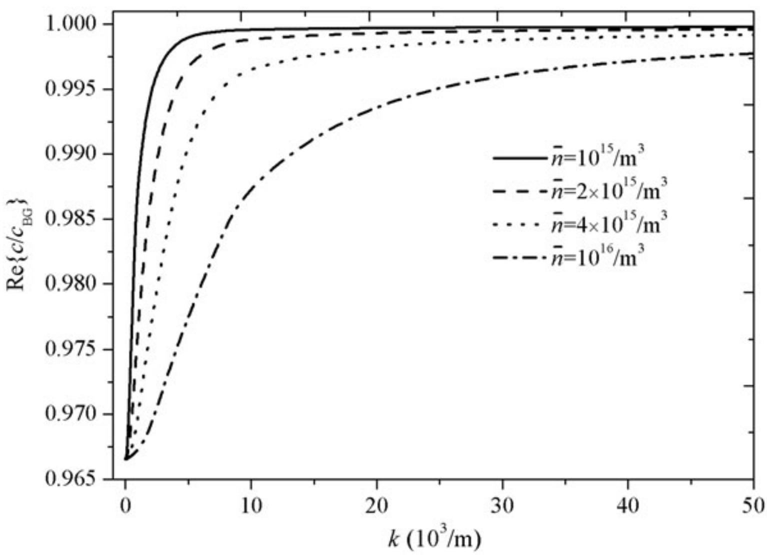

Figure 2a. Effect of the steady-state carrier density $\bar{n}$ on wave speed for $\bar{E}_2 = 0$.

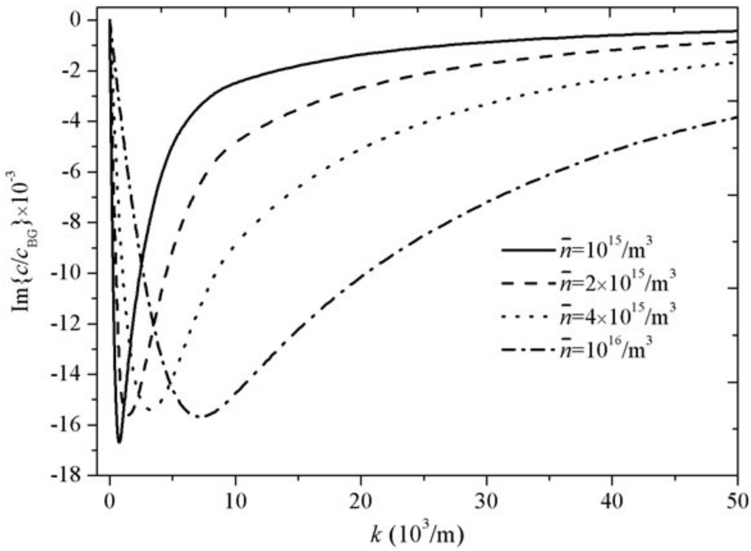

Figure 2b. Effect of the steady-state carrier density $\bar{n}$ on wave attenuation for $\bar{E}_2 = 0$.

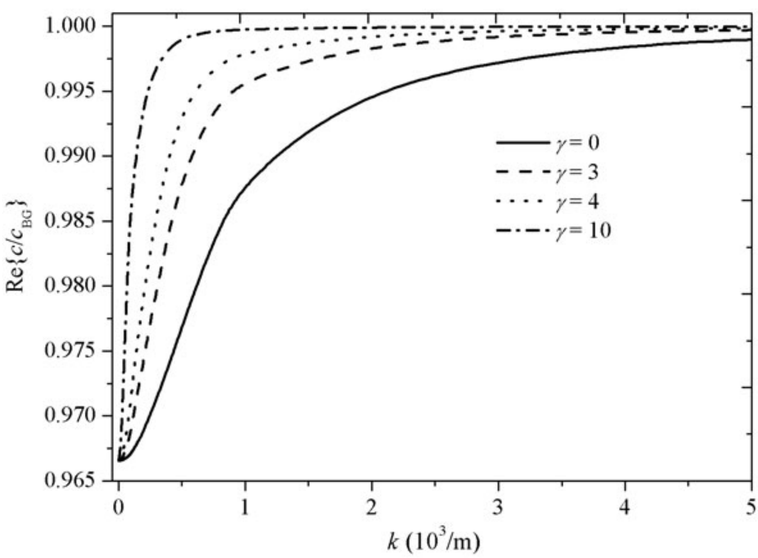

Figure 3a. Effect of the biasing electric field on wave speed.

not helpful to conduction and therefore the effect of semiconduction is weak. At the long wave limit, the wave speed approaches v_{BG} for the same reason. When $\bar{n}$ increases or there are more carriers, the effect of semiconduction becomes stronger as expected.

The normalized imaginary part of c which describes wave attenuation is shown in Figure 2b. It is always negative, showing wave amplitude attenuation instead of growth.

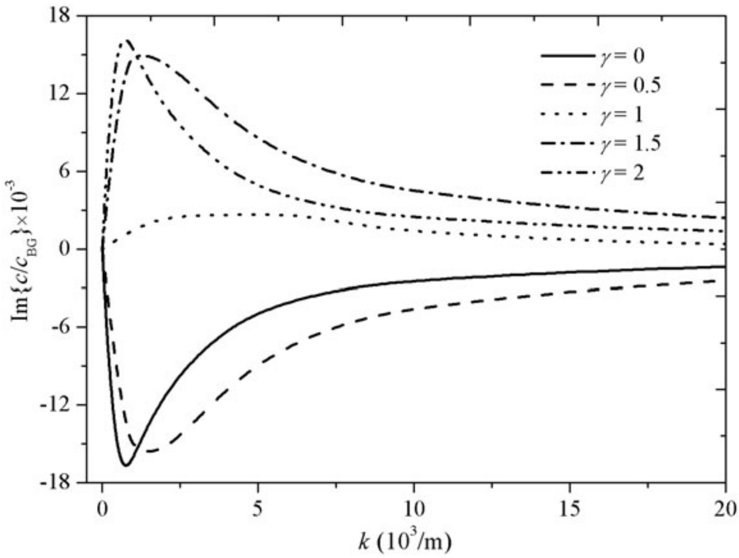

Figure 3b. Effect of the biasing electric field on wave attenuation.

For short waves, the attenuation due to semiconduction is weak like in the case of the wave speed shown in Figure 2a and is for the same reason. Somewhere between short and long waves, there is a maximal attenuation at a particular value of *k*. For very long waves ($k \to 0$), the attenuation becomes small again. These are not easy to explain and were also observed in [18] for waves in structures with components of both non-piezoelectric semiconductors and piezoelectric dielectrics.

5.2. *Wave Amplification by the biasing electric field*

Figures 3a and b show the effect of the biasing electric field $\bar{E}_2$ on the complex wave speed when $\bar{n} = 10^{15}$. $\gamma = \mu\bar{E}_2/c_{BG}$ is the ratio between the carrier drift speed under the biasing electric field and the speed of Bleustein–Gulyaev wave. When $\bar{E}_2$ is present, the effect of semiconduction on the wave speed is still qualitatively similar to the case when $\bar{E}_2 = 0$ as shown in Figure 3a. However, the effect of $\bar{E}_2$ on wave attenuation has qualitatively different behaviours roughly divided by the case of $\gamma = 1$. When γ is roughly less than 1, we still have wave attenuation as in the case shown in Figure 2b. When γ is roughly larger than 1, the attenuation changes its sign and we in fact have wave growth. This is the important phenomenon of wave amplification by a biasing electric field in a piezoelectric semiconductor [6–9].

6. Conclusion

A shear-horizontal surface wave is shown to exist in a half-space of a piezoelectric semiconductor of crystals of class 6 mm. It generalizes the well-known Bleustein–Gulyaev wave in a non-conducting piezoelectric half-space. The semiconduction affects the wave speed and causes both dispersion and wave attenuation. The effect of semiconduction is stronger for higher carrier densities as expected. It diminishes for short waves at high frequencies. The surface wave can be amplified by a biasing electric field in the wave propagation direction. The amplification occurs roughly when the carrier drift speed under the biasing electric field exceeds the surface wave speed.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This work was supported by the National Natural Science Foundation of China [grant number 11272247]; the National 111 Project of China [grant number B06024].

References

[1] B.A. Auld (ed.), *Acoustic Fields and Waves in Solids*, Wiley, New York, 1973.
[2] A.R. Hutson and D.L. White, J. Appl. Phys. 33 (1962) p.40.
[3] G. Weinreich, T.M. Sanders and H.G. White, Phys. Rev. 114 (1959) p.33.
[4] D.R. Dietz, L.J. Busse and M.J. Fife, IEEE Trans. Ultrason. Ferroelectr. Freq. Control 35 (1988) p.146.
[5] J.N. Sharma, K.K. Sharma and A. Kumar, J. Mech. Mater. Struct. 6 (2011) p.791.
[6] D.L. White, J. Appl. Phys. 33 (1962) p.2547.
[7] J.S. Yang and H.G. Zhou, Acta Mech. 172 (2004) p.113.
[8] S. Ghosh and P. Khare, Indian J. Pure Appl. Phys. 44 (2006) p.183.
[9] M. Willatzen and J. Christensen, Phys. Rev. B 89 (2014) p.041201.
[10] J.R. Schuelein, K. Mueller, M. Bichler, G. Koblmueller, J.J. Finley, A. Wixforth and H.J. Krenner, Phys. Rev. B 88 (2013) p.085307.
[11] S. Buyukkose, A. Hernandez-Minguez, B. Vratzov, C. Somaschini, L. Geelhaar, H. Riechert, W.G. van der Wiel and P.V. Santos, Nanotechnology 25 (2014) p.135204.
[12] Z.L. Wang, Adv. Mater. 19 (2007) p.889.
[13] O. Graton, G. Poulin-Vittrant, L.P. Tran Huu Hue and M. Lethiecq, Adv. Appl. Ceram. 112 (2013) p.85.
[14] K. Yin, H.Y. Lin, Q. Cai, Y. Zhao, S.T. Lee, F. Hu and M.W. Shao, Nanoscale 5 (2013) p.12330.
[15] P. Hiralal, H.E. Unalan and G.A. Amaratunga, Nanotechnology 23 (2012) p.194002.
[16] B. Kumar and S.W. Kim, Nano Energy 1 (2012) p.342.
[17] B. Collet, J. Acoust. Soc. Am. 123 (2008) p.3694.
[18] J.N. Sharma, K.K. Sharmar and A. Kumar, World J. Mech. 1 (2011) p.247.
[19] N. Rani, M. Yadav and Y.K. Mathur, Int. J. Appl. Eng. Res. 9 (2014) p.59.
[20] J.L. Bleustein, Appl. Phys. Lett. 13 (1968) p.412.
[21] Y.V. Gulyaev, J. Exp. Theor. Phys. Lett. 9 (1969) p.37.
[22] J. Wauer and S. Suherman, Int. J. Eng. Sci. 35 (1997) p.1387.
[23] H.F. Tiersten and T.L. Sham, IEEE Trans. Ultrason. Ferroelectr. Freq. Control 45 (1998) p.1.
[24] L.F. Qin, Q.M. Chen, H.B. Cheng, Q. Chen, J.F. Li and Q.M. Wang, J. Appl. Phys. 110 (2011) p.094511.
[25] D.H. Navon (ed.), *Semiconductor Microdevices and Materials*, CBS College Publishing, New York, 1986.
[26] D.J. Leary, J.O. Barnes and A.G. Jordan, J. Electrochem. Soc. 129 (1982) p.1382.
[27] C.A. Jackson, P. Moetakef, S.J. Allen and S. Stemmer, Appl. Phys. Lett. 100 (2012) p.232106.