A simplistic analytical unit cell based model for the effective thermal conductivity of high porosity open-cell metal foams

2013 J. Phys. D: Appl. Phys. 46 255302

(http://iopscience.iop.org/0022-3727/46/25/255302)

J. Phys. D: Appl. Phys. **46** (2013) 255302 (6pp) doi:10.1088/0022-3727/46/25/255302

A simplistic analytical unit cell based model for the effective thermal conductivity of high porosity open-cell metal foams

X H Yang[1], J J Kuang[1], T J Lu[2], F S Han[3] and T Kim[4]

[1] School of Energy and Power Engineering, Xi'an Jiaotong University, Xi'an 710049, People's Republic of China
[2] State Key Laboratory for Mechanical Structure Strength and Vibration, Xi'an Jiaotong University, Xi'an 710049, People's Republic of China
[3] Institute of Solid Physics, Chinese Academy of Science, Hefei, 230031, People's Republic of China
[4] School of Mechanical Engineering, University of the Witwatersrand, Private Bag 3, Wits 2050, Johannesburg, South Africa

E-mail: tong.kim@wits.ac.za (T Kim)

Received 2 February 2013, in final form 8 May 2013
Published 10 June 2013
Online at stacks.iop.org/JPhysD/46/255302

Abstract
We present a simplistic yet accurate analytical model for the effective thermal conductivity of high porosity open-cell metal foams saturated in a low conducting fluid (air). The model is derived analytically based on a realistic representative unit cell (a tetrakaidecahedron) under the assumption of one-dimensional heat conduction along highly tortuous-conducting ligaments at high porosity ranges ($\varepsilon \geqslant 0.9$). Good agreement with existing experimental data suggests that heat conduction along highly conducting and tortuous ligaments predominantly defines the effective thermal conductivity of open-cell metal foams with negligible conduction in parallel through the fluid phase.

(Some figures may appear in colour only in the online journal)

1. Introduction

The distinctive properties, relatively low manufacturing cost, ultra-low density, and high surface area-to-volume ratio of metal foams with open cells (or open-cell metal foams, figure 1(*a*)) have led to their utilization in a variety of engineering applications such as microelectronics cooling [1], fuel cells [1, 2] and compact heat exchangers [3–6]. Numerous studies have therefore evaluated effective thermo-physical properties of these foams, including effective thermal conductivity as a key factor in the thermal analysis of metal foam-cored heat exchanger systems [5, 6].

After Maxwell's pioneering work [7], a multitude of analytical studies have been dedicated to estimate the effective thermal conductivity of porous media having various topologies [8–11], based on some assumptions such as series or parallel configuration of solid and fluid phases, random distribution of each phase [12, 13] and symmetrical distribution of the two phases [14]. However, these models have shown to be inapplicable to open-cell metal foams. Such inapplicability may be due to the idealized topology of porous media adopted during the derivation.

Some recent studies have attempted to develop effective thermal conductivity models accounting for realistic topologies for open-cell metal foams (figure 1(*b*)). Calmidi and Mahajan [15] and Bhattacharya *et al* [16] took a two-dimensional unit cell having a hexagonal honeycomb shape with square [15]/circular [16] nodes at each joint (figure 1(*b*)), representing open-cell aluminum (Al) foams saturated in a fluid phase (e.g., air). Employing these unit cells, 'semi'-analytical models for estimating effective thermal conductivity were proposed, but essentially requiring a fitting parameter (the ratio of ligament length to the side length of square node). These models were extended to

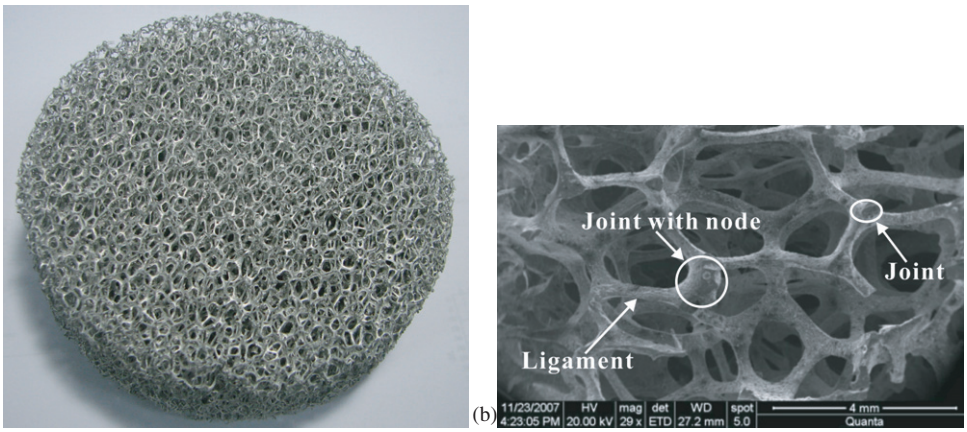
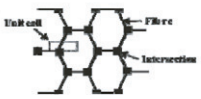
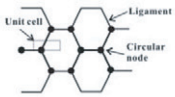
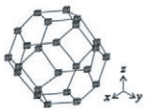


Figure 1. High porosity aluminum (Al) foam with open cells ($\varepsilon = 0.92$). (*a*) Photograph of a cylindrical block of Al foam. (*b*) Scanning electronic microscope (SEM) image of Al foam.

Table 1. Fundamental unit cell models specifically developed for open-celled foam-type porous media.

Model	Unit cell	Expression	Comments
Two-dimensional hexagonal unit cell with square node (Calmidi and Mahajan[a])		$k_e = \left(\frac{2}{\sqrt{3}}\right)\left(\frac{r\left(\frac{b}{L}\right)}{k_f + \left(1+\frac{b}{L}\right)\frac{(k_s-k_f)}{3}} + \frac{(1-r)\left(\frac{b}{L}\right)}{k_f + \frac{2}{3}\left(\frac{b}{L}\right)(k_s-k_f)} + \frac{\frac{\sqrt{3}}{2}-\frac{b}{L}}{k_f + \frac{4r}{3\sqrt{3}\left(\frac{b}{L}\right)}(k_s-k_f)}\right)^{-1}$ where $\frac{b}{L} = \frac{-r+\sqrt{r^2+\frac{2}{\sqrt{3}}(1-\varepsilon)\left(2-r\left(1+\frac{4}{\sqrt{3}}\right)\right)}}{\frac{2}{3}\left(2-r\left(1+\frac{4}{\sqrt{3}}\right)\right)}$ and $r = 0.09$	- Two-dimensional semi-analytical model - Unrealistic micro structure: hexagonal honeycomb shape with square nodes joined by the solid ligaments - Adjustable parameter (r = 0.09) determined through fitting the experimental data
Two-dimensional hexagonal unit cell with circular node (Bhattacharya *etal*[b])		$k_e = \left(\frac{2}{\sqrt{3}}\right)\left(\frac{t/L}{k_f + \frac{1}{3}(k_s-k_f)} + \frac{\sqrt{3}/2 - t/L}{k_f}\right)^{-1}$ where $\frac{t}{L} = \frac{-\sqrt{3}-\sqrt{3+(1-\varepsilon)(\sqrt{3}-5)}}{1+\frac{1}{\sqrt{3}}-\frac{8}{3}}$ and $r = 0.19$	-Two-dimensional semi-analytical model -Unrealistic structure: hexagonal honeycomb shape with circular nodes joined by the solid ligaments -Adjustable parameter (r = 0.19) determined through fitting the experimental data
Cubic cell (Peak *etal*[c])		$k_e = k_s t^2 + k_f(1-t)^2 + \frac{2t(1-t)k_f k_s}{k_s(1-t)+k_f t}$ where $t = \frac{1}{2} + \cos\left(\frac{1}{3}\cos^{-1}(2\varepsilon - 1) + \frac{4\pi}{3}\right)$	-Three-dimensional analytical model -Unrealistic micro structure: twelve cubic ligaments contained in a cube -Relatively good agreement with experimental data
Tetradecahedton unit cell (Boomsma and Poulikakos[d])		$k_e = \frac{\sqrt{2}}{2(R_A + R_B + R_C + R_D)}$ where $R_A = \frac{4d}{\left(2e^2+\pi d(1-e)\right)k_s + \left(4-2e^2-\pi d(1-e)\right)k_f}$ $R_B = \frac{(e-2d)^2}{(e-2d)e^2 k_s + (2e-4d-(e-2d)e^2)k_f}$ $R_C = \frac{\left(\sqrt{2}-2e\right)^2}{2\pi d^2\left(1-2e\sqrt{2}\right)k_s + 2\left(\sqrt{2}-2e-\pi d^2\left(1-2e\sqrt{2}\right)\right)k_f}$ $R_D = \frac{2e}{e^2 k_s + \left(4-e^2\right)k_f}$, $d = \sqrt{\frac{\sqrt{2}\left(2-(5/8)e^3\sqrt{2}-2\varepsilon\right)}{\pi(3-4e\sqrt{2}-e)}}$, and e = 0.339	-Three-dimensional semi-analytical model -Realistic micro structure: tetrakaidecahedron geometry with cubic nodes in intersection of ligaments -Adjustable parameter (cubic node size) determined through fitting experimental data -Only valid in the porosity range ε<0.98

[a] See [15].
[b] See [16].
[c] See [18].
[d] See [19].

accommodate the three-dimensional topology of open-cell Al foams by Boomsma and Poulikakos [17] who adopted a tetrakaidecahedron unit cell. For simplicity in their study, the cylindrical ligaments and cubic nodes at each joint were assumed. Although this tetrakaidecahedron unit cell model could provide a good estimate of effective thermal conductivity for open-cell Al foams, the model has a quite complex form (tabulated in table 1) and again essentially requires a geometrical fitting factor i.e. the ratio of the cubic node to the ligament length, making the model 'semi'-analytical.

The above-mentioned semi-analytical models predict that the effective thermal conductivity (k_e/k_s) of open-cell metal (e.g., Al) foams is significantly reduced with an increase in porosity whilst being in good agreement with experimental data [15, 18–20] as plotted in figure 2. At high porosity ranges ($\varepsilon \geqslant 0.9$), the effective thermal conductivity ranges from 10% to 30% of that of the solid phase, corresponding inversely to the porosity.

Some empirical correlations of the effective thermal conductivity of open-cell Al foams saturated in air have also been reported as

(i) from experimental data fitting [16],

$$\frac{k_e}{k_s} = 0.350(1-\varepsilon), \tag{1a}$$

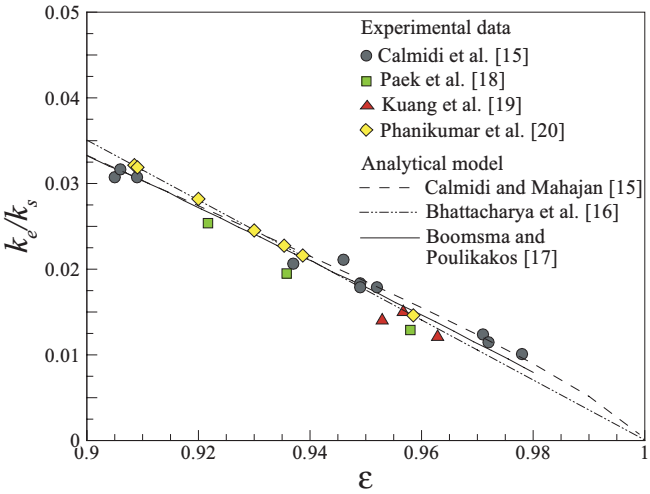


Figure 2. Dependence of effective thermal conductivity on porosity at high porosity ranges ($\varepsilon > 0.9$), obtained experimentally and analytically for open-cell Al foams [15–20].

(ii) through the analogy between electrical and thermal conductivity [3],

$$\frac{k_e}{k_s} = 0.346(1-\varepsilon), \tag{1b}$$

(iii) by fitting experimental data included in figure 2,

$$\frac{k_e}{k_s} = 0.319(1-\varepsilon) \qquad \text{with } R^2 = 0.97. \tag{1c}$$

These correlations have a simple form resembling the classical parallel model [15–17]. However, the physical basis of these empirical correlations in terms of (i) taking the form of the parallel model and (ii) having a similar empirical coefficient ranging from 0.319 to 0.350 has not yet been provided.

This study aims to squarely address this deficiency by introducing a purely analytical model for the effective thermal conductivity of open-cell metal foams. To this end, heat conduction in a three-dimensional tetrakaidecahedron unit cell is considered, assuming one-dimensional conduction along highly tortuous-conducting ligaments with negligible conduction through the fluid phase (air) filling the voids of metal foams. It is anticipated that a fully analytical expression demonstrates physical insight into how the effective thermal conductivity is influenced as the porosity of open-cell metal foams is varied in high porosity ranges ($\varepsilon \geqslant 0.9$).

2. Description of generic and particular analytical models

2.1. Generic expression assuming one-dimensional heat conduction

Consider a high porosity medium with open cells distributed either randomly or periodically, and fully saturated in a low- (or non-) conducting fluid phase as illustrated in figure 3. Let k_e denote the effective thermal conductivity of an arbitrary unit cell; k_s and k_f are the thermal conductivity of the solid ligaments and the fluid saturating the unit cell. Following

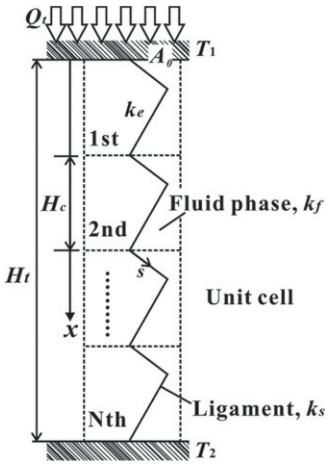


Figure 3. A simple illustration of a two-phase porous medium having the effective thermal conductivity of k_e where interconnected unit cells can be either randomly or periodically distributed and $k_s \gg k_f$.

Fourier's law, the total amount of heat subjecting to a control volume is expressed as

$$Q_t = k_e A_0 \frac{\Delta T}{H_c}, \tag{2}$$

where A_0 is the heat transfer area of the unit cell, H_c is the thickness of the unit cell and ΔT is the temperature difference across the unit cell along the x-axis.

The total amount of heat transferred to the unit cell is conducted along the solid ligaments (k_s) and the fluid (k_f) in parallel i.e. $Q_t = Q_s + Q_f$. According to Fourier's law, one-dimensional conduction along the ligaments may be expressed as

$$Q_s = -k_s A_s \frac{dT}{ds}. \tag{3}$$

To solve equation (3), the method of separation of variables is applied where A_s (the cross-sectional area of each ligament) is a function of ds independent of dT, yielding

$$\frac{Q_s}{A_s} ds = -k_s \, dT, \tag{4}$$

where s is the axis along the ligaments, strongly dependent on the topology of the porous medium of interest. Integration of equation (4) leads to:

$$Q_s \int_0^{H(s)} \frac{1}{A_s} ds = -k_s \, T|_0^{H_c}. \tag{5}$$

It should be noted that the heat Q_s along the tortuous ligaments is assumed to be constant (through the ligaments of the same cross-sectional area). Mathematical manipulation of equation (5) yields

$$Q_s = k_s \Delta T \Big/ \left(\int_0^{H(s)} \frac{1}{A_s} ds \right). \tag{6}$$

Comparing equation (2) and equation (6) reduces the effective thermal conductivity normalized by the solid phase's thermal

conductivity (k_e/k_s) to

$$\frac{k_e}{k_s} = \left(\frac{H_c}{A_0}\right) \Big/ \left(\int_0^{H(s)} \frac{1}{A_s}\, ds\right), \tag{7}$$

where the conduction of heat through the fluid phase is neglected. The final expression, equation (7) can only be analytically solved if topological parameters associated with individual porous media are prescribed.

2.2. Particular expression for a tetrakaidecahedron unit cell

To derive a particular analytical solution for the general equation (equation (7)), a proper selection of a representative unit cell is a necessity [14–19]. Table 1 lists some unit cell models recently developed for open-cell foam-type porous media. The two-dimensional unit cell models proposed by Calmidi and Mahajan [15] and Bhattacharya *et al* [16] cannot reflect the topology of real metal (e.g., Al) foams, also requiring an empirical parameter (a node size).

A more complex but realistic unit cell for Al foams was proposed by Boomsma and Poulikakos, consisting of six square and eight hexagonal faces [17]: a tetrakaidecahedron, which is observable during the foaming process [21, 22]. The foaming gas in the molten slurry moves freely and finally forms tetrakaidecahedron cells to keep minimal surface energy [23, 24]. The actual topology and its representative unit cell of open-cell Al foams are separately shown in figures 1(*b*) and 4. The tetrakaidecahedron cell contained in a cube unit cell consists of six square and eight hexagon faces with a ligament length of a and a ligament thickness of t. It should be noted that the size of the nodes is, neglected, assuming its minimal influence (to be discussed later). For simplicity, one sixteenth out of the single tetrakaidecahedron cell – a cube with the dimension of $\sqrt{2}a$ (length) × $\sqrt{2}a$ (width) × $\sqrt{2}a/2$ (height) is only considered, which contains two entire ligaments with a length of a and a thickness of t as well as a partial ligament having a in length and $t/2$ in thickness.

The relative density of open-cell foams having the idealized tetrakaidecahedron unit cell can be expressed as a function of ligament length, a and thickness, t (for the square cross-section) as

$$\rho^* = 1 - \varepsilon = \frac{3\sqrt{2}}{4}\frac{t^2}{a^2}. \tag{8}$$

The variation of the heat transfer area integrated along the s-direction in equation (7) can be calculated as

$$\int_0^{H(s)} \frac{1}{A_s}\, ds = \frac{a}{t^2}. \tag{9}$$

The total heat transfer area and unit cell length can be determined as

$$A_0 = 2a^2, \tag{10}$$

$$H_c = \frac{\sqrt{2}a}{2}. \tag{11}$$

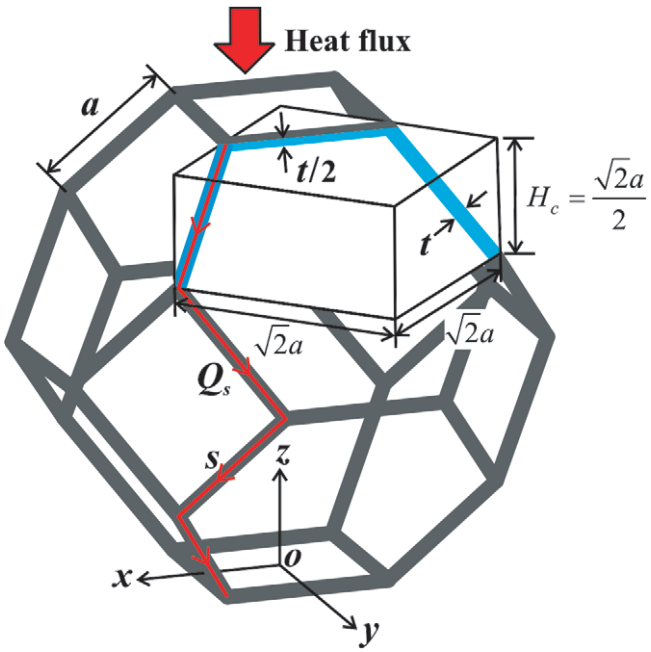


Figure 4. An open-cell Al foam modelled with a tetrakaidecahedron unit cell having a uniform thickness of square ligaments, t and ligament length, a. The cube denotes a control volume for one sixteenth of a single tetrakaidecahedron unit cell.

The present one-dimensional conduction method gives a particular expression for the effective thermal conductivity of open-cell metal foams as

$$\frac{k_e}{k_s} = \left(\frac{H_c}{A_0}\right) \Big/ \left(\int_0^{H(s)} \frac{1}{A_s}\, ds\right) = \frac{\sqrt{2}}{4}\frac{t^2}{a^2}. \tag{12}$$

Substitution of the porosity (equation (8)) into equation (12) yields the final expression for the effective thermal conductivity of the open-cell metal foams having the idealized tetrakaidecahedron unit cell as

$$\frac{k_e}{k_s} = \frac{1}{3}(1 - \varepsilon). \tag{13}$$

The expression of the effective thermal conductivity k_e/k_s in equation (13) has a remarkably simple form—a product of the porosity and the coefficient of 1/3.

3. Discussion of results

3.1. Dependence of effective thermal conductivity on porosity

The effective thermal conductivity (k_e/k_s) of open-cell metal foams strongly depends on porosity. Figure 5 shows the substantial reduction of the effective thermal conductivity as the porosity is increased in high porosity ranges ($\varepsilon \geqslant 0.9$). The present analytical model predicts accurately the trend of experimental data [15, 18, 24] where the deviation is occurred within ±3.5%. However, a further decrease in porosity ($\varepsilon < 0.9$) will result in the thickening of the ligaments for a given pore size. As a result, conduction along the thickened ligaments violates the one-dimensional conduction assumption, which consequently over-estimates the effective

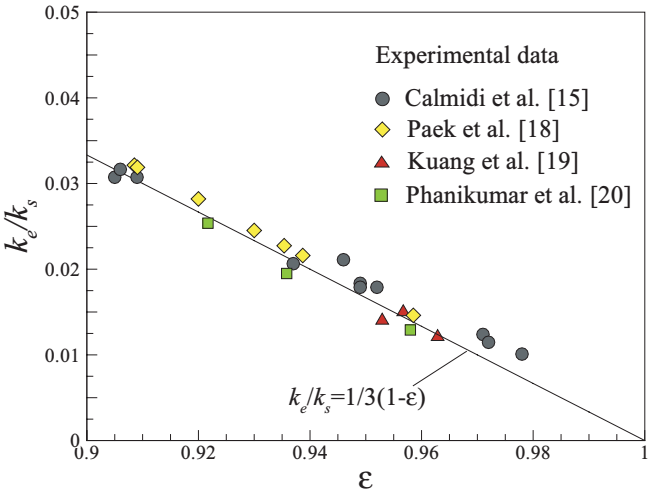


Figure 5. Comparison of the effective thermal conductivity predicted by the present model (equation (13)) with experimental data [15, 18–20].

thermal conductivity of open-cell metal foams. In other words, two- and three-dimensional conduction within the ligaments yields a more tortuous conduction path leading to the lowered effective thermal conductivity (not covered in the present study).

The coefficient 1/3 (or ∼0.33) in the analytical model (equation (13)) is the reciprocal of thermal tortuosity, stating the elongated heat transfer length along the tortuous-thin ligaments [25, 26]. In open-cell metal foams especially made of highly conducting metals e.g., aluminum, heat is conducted in parallel (via foam ligaments and saturating fluid). The negligible conduction through the fluid phase may be justified when a thermal conductivity ratio between the solid and fluid phases is large (e.g., $k_s/k_f > 8000$ for air-saturated Al foam). The one-dimensional conduction along the ligaments may be ensured by a large slenderness ratio of the ligaments (a/t) e.g., $a/t \sim 7.2$ for Al foams with $\varepsilon = 0.913$ (10 pores per inch, PPI) and 40 PPI [27].

3.2. Pore and node sizes

The present model (equation (13)) explicitly expresses the dependence of the effective thermal conductivity on porosity whereas a possible effect of pore size is not included. Previous studies [15, 16, 18, 28, 29] have univocally argued that the pore size plays little part in the effective thermal conductivity of open-cell metal foams saturated by air, under 'typical' conditions. However, under 'certain' situations, its effect becomes substantial in determining the effective thermal conductivity as:

(i) for a given high porosity, if heat transfer via a fluid phase is considerable, natural convection within the fluid phase saturating a metal foam cannot be neglected [27]. In this case, larger pores allow larger space for natural convection to be enhanced. As a result, the effective thermal conductivity of the metal foam is increased, and

(ii) if a representative (or average) temperature is high enough (>400 K), the extinction coefficient (the sum of absorption and scattering coefficients) tends to be decreased as the pore size is increased. Consequently, the 'penetration thickness' becomes thicker than that associated with smaller pores. Thicker penetration thickness implies that more heat can be directly transferred by thermal radiation, reducing the total thermal resistance and increasing the effective thermal conductivity [30].

Open-cell metal foams consist of a complex array of interconnected ligaments. Irregular nodes exist at each joint [15–17] where a node refers to an abnormally big joint (thicker than ligament). Most of the joints are slightly thicker than ligaments whose relative size to the ligament thickness strongly depends on individual manufacturing routes. For example, an average node-to-ligament thickness ratio measured from the SEM image analysis on open-cell Al foams ($\varepsilon = 0.92$) is estimated to be about 1.3. Temperature distortion in the nodes (in figure 1(*b*)), which leads to an elongated heat transfer length (or thermal tortuosity), is therefore expected to be marginal (or negligible) as assumed during the derivation of the present analytical model. In comparison, this ratio for square nodes [15], circular nodes [16] and cubic nodes [17] was reported to be approximately 11.1, 5.3 and 5.8, respectively (by fitting the semi-analytical solutions against experimental data [15–17]), which is unrealistic. As varying the pore size, the average node-to-ligament thickness ratio tends to vary. However, it has been observed that within the considered high porosity ranges ($\varepsilon \geqslant 0.9$), the variation of this ratio is insignificant [15, 16, 18, 28, 29].

3.3. Ligament's cross-sectional shape

During the model derivation, ligaments having a square cross-section were assumed. Other cross-sectional shapes such as circle and triangle may also be assumed. For a given high porosity and fixed pore size, there is no influence upon the effective thermal conductivity of open-cell metal foams. This is because the cross-sectional 'area' of the ligaments remains unchanged regardless of the cross-section shapes i.e. $A_{\text{square}} = A_{\text{triangle}} = A_{\text{circle}} = t^2$. The cross-sectional area of different shaped ligaments takes the form:

$$A_{\text{square}} = t^2, \tag{14a}$$

$$A_{\text{triangle}} = \frac{\sqrt{3}}{4} t_1^2, \tag{14b}$$

$$A_{\text{circle}} = \frac{\pi}{4} t_2^2. \tag{14c}$$

where $t_1 \sim 1.519t$ and $t_2 \sim 1.128t$ for the same cross-section area as the square ligament.

It should be noted that the surface area of the ligaments of the three different shapes differs depending on the ligament shape although other parameters e.g., pore size, relative density are invariant. However, the surface area does not contribute to the effective thermal conductivity as no heat transfer between the solid ligaments and the saturating fluid and no radiation from the solid ligaments were assumed. On the other hand, the surface area will play a significant role in other heat transfer modes e.g., radiation and forced convection, which are not considered in the present model.

3.4. Other unit cell topologies

The effective thermal conductivity of open-cell metal foams strongly depends on the selected unit cell as the conduction distance (or thermal tortuosity) varies accordingly. Adopting the present model (equation (7)), different representative unit cells yield:

(i) for two-dimensional hexagonal honeycomb cell [15, 16],

$$\frac{k_e}{k_s} = \frac{1}{2}(1-\varepsilon), \tag{15a}$$

(ii) for three-dimensional cube cell [18],

$$\frac{k_e}{k_s} = \frac{2}{3}(1-\varepsilon). \tag{15b}$$

Although these two models (equations (15*a*) and (15*b*)) have a form resembling the parallel model, they cannot provide a satisfactory estimate of the effective thermal conductivity due to the different extent of the elongated heat transfer distance (thermal tortuosity). The effective thermal conductivity can be predicted based only on a unit cell which accurately describes the actual three-dimensional topology of high porosity open-cell metal foams.

4. Conclusions

A simple yet physically sound model capable of analytically estimating the effective thermal conductivity of high porosity metal foams with open cells is presented. With one-dimensional heat conduction assumed, a realistic tetrakaidecahedron unit cell was selected for representing the topology of typical open-cell metal foams. Results demonstrate that heat conduction via fluid phase can be neglected due to a large ratio of stagnant thermal conductivities between solid phase (ligaments) and fluid phase such as air-saturating aluminum foams. The present analytical model that does not require any empirical factors provides the physical insight into the existing empirical correlations and the physical evidence of the empirical coefficient being approximately 0.33 in the parallel model for high porosity open-cell metal foams.

Acknowledgment

This work was financed by the National 111 Project of China (B06024), the National Basic Research Program of China (2011CB610305), the Major International Joint Research Program of China (11120101002) and the National Natural Science Foundation of China (51206128).

References

[1] Kaviany M 1995 *Principles of Heat Transfer in Porous Media* (New York: Springer) p 125
[2] Dukhan N, Picon-Feliciano R and Alvarez-Hernandez A R 2006 *J. Heat Transfer* **128** 784
[3] Ozmat B, Leyda B and Benson B 2004 *Mater. Manuf. Processes* **19** 839
[4] Kim S Y, Paek J W and Kang B H 2000 *J. Heat Transfer* **122** 572
[5] Hunt M L and Tien C L 1988 *Int. J. Heat Mass Transfer* **31** 301
[6] Mahjoob S and Vafai K 2008 *Int. J. Heat Mass Transfer* **51** 3701
[7] Maxwell J C 1873 *A Treatise on Electricity and Magnetism* (Oxford: Clarendon Press) p 365
[8] Progelhof R C, Throne J L and Reutsch R R 1976 *Polym. Eng. Sci.* **16** 615
[9] Tsotasas E and Martin H 1987 *Chem. Eng. Process.* **22** 19
[10] Hashin Z and Shtrikman S 1962 *J. Appl. Phys.* **33** 3125
[11] Russell H W 1935 *J. Am. Ceram. Soc.* **18** 1
[12] Landauer R 1952 *J. Appl. Phys.* **23** 779
[13] Kirkpatrick S 1973 *Rev. Mod. Phys.* **45** 574
[14] Hsu C T, Cheng P and Wong K W 1994 *Int. J. Heat Mass Transfer* **37** 2751
[15] Calmidi V V and Mahajan R L 1999 *J. Heat Transfer* **121** 466
[16] Bhattacharya A, Calmidi V V and Mahajan R L 2002 *Int. J. Heat Mass Transfer* **45** 1017
[17] Boomsma K and Poulikakos D 2001 *Int. J. Heat Mass Transfer* **44** 827
[18] Paek J W, Kang B H, Kim S Y and Hyun J M 2000 *Int. J. Thermophys.* **21** 453
[19] Kuang J J, Yang X H, Lu T J and Kim T 2012 A unit cell analysis for the open-cell metal foams: a review *XJTU Technical Report* 2012-002
[20] Phanikumar M S and Mahajan M L 2002 *Int. J. Heat Mass Transfer* **45** 3781
[21] Weaire D and Hutzler S 1999 *The Physics of Foam* (Oxford: Clarendon Press) p 153
[22] Thompson D 1995 *On Growth and Form* (*Canto*) (Cambridge: Cambridge University Press) p 121
[23] Thomson W 1887 *Phil. Mag.* **5** 503
[24] Gibson L J and Ashby M F 1999 *Cellular Solids: Structure and Properties* 2nd edn (Cambridge: Cambridge University Press) p 15
[25] Yang X H, Lu T J and Kim T 2013 *Theor. Appl. Mech. Lett.* **3** 021011
[26] Yang X H, Lu T J and Kim T 2013 *J. Phys. D: Appl. Phys.* **46** 125305
[27] Calmidi V V 1998 *D. Phil. Dissertation* University of Colorado
[28] Takegoshi E, Hirasawa Y, Matsuo J and Okui K 1992 *Trans. Japan. Soc. Mech. Eng.* **58** 879
[29] Dul'nev G N 1965 *J. Eng. Phys.* **9** 275
[30] Zhao C Y, Lu T J, Hodson H P and Jackson J D 2004 *Mater. Sci. Eng.* A **367** 123