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An analytical model for permeability of isotropic porous media

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ABSTRACT

We demonstrate that permeability of isotropic porous media e.g., open-cell foams can be analytically presented as a function of two morphological parameters: porosity and pore size. Adopting a cubic unit cell model, an existing tortuosity model from the branching algorithm method is incorporated into a generalized permeability model. The present model shows that dimensionless permeability significantly increases as the porosity of isotropic porous media and unifies the previously reported data in a wide range of porosity ($\varepsilon = 0.55$–0.98) and pore size ($D_p = 0.254$ mm–5.08 mm).

1. Introduction

The understanding of transport phenomena in natural and engineering porous media paves the foundation for industrial and environmental processes, covering a broad range of disciplines e.g., geology, engineering, chemistry, and physics. Their engineering application includes catalysis, separation and filtration, cement chemistry, fuel cell, and heat exchanger [1–4]. Amongst various porous media, open-cell metallic foams have shown distinctive properties: relatively low manufacturing costs, ultra-low density, and high surface area-to-volume ratio. They have been, therefore, utilized in a variety of engineering applications such as microelectronics cooling [5], fuel cells [5,6] and compact heat exchangers [6–10].

Permeability is a property that describes the easiness of fluid flow through a void space in open cellular porous media. It depends on the intrinsic topology of individual porous media. Permeability (denoted by K) appears in Darcy's law which governs flow through porous media in creeping flow regime [5], expressed as:

$$-\frac{\mathrm{d}p}{\mathrm{d}x} = \frac{\mu u}{K} \tag{1}$$

where $\mathrm{d}p/\mathrm{d}x$ is the pressure drop along the nominal flow direction (e.g., x-direction), μ is the dynamic viscosity, and u is the mean velocity along the x-direction. For higher-velocity regime, this Darcy's law was extended by Forchheimer [11], considering the inertial effect for the steady-state, uni-directional pressure drop in homogeneous and isotropic porous media which are fully saturated by a Newtonian incompressible fluid:

$$-\frac{\mathrm{d}p}{\mathrm{d}x} = \frac{\mu u}{K} + \frac{\rho f}{\sqrt{K}}u^2 \tag{2}$$

Permeability is coupled with the inertial coefficient (f). Scale analysis shows that permeability has a dimension of the square of a length scale.

With known permeability for a particular porous medium, the control of fluid flow, heat transfer enhancement, optimal flow analysis as well as practical designs are facilitated. Due to such importance, numerous attempts have been made to estimate permeability of various porous media [12–24]. Whilst most of earlier models were developed based on the curve-fitting of experimental or numerical data [25–30], there are some analytical models for porous media with specific and well-defined pore topologies, requiring no adjustable factors but with highly idealized topologies such as one-/two-dimensional structure and ordered arrangement [12,13,15]. As listed in Table 1, these analytical models are complex in form, indirect, and have limited valid porosity ranges [12,13,30].

Amongst these analytical models, an analytical permeability model proposed by Du Plessis et al. [31] and Fourie and Du Plessis [32] is for open-cell foams, via a microstructure based approach through a cubic unit cell modeling. The results showed promising agreement in a small pore size range (e.g., 45PPI–100PPI) and a very narrow porosity range (0.973–0.978) for both Darcy and Forchheimer flow regimes. The extension of the model was

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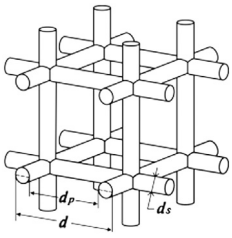
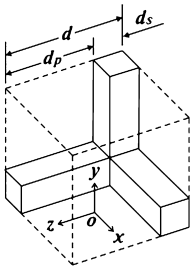
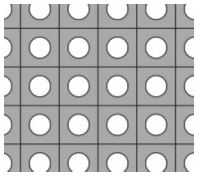
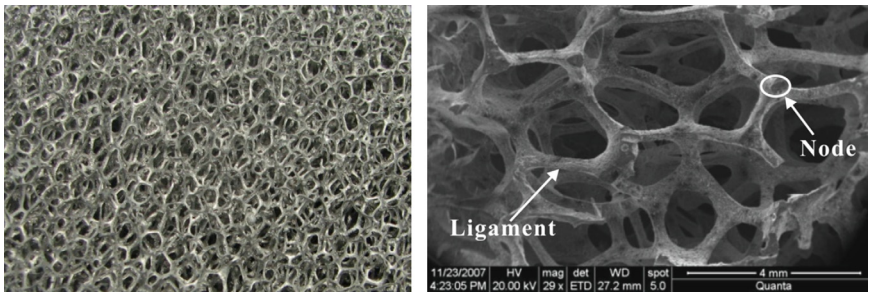
Table 1
Fundamental unit cell models specifically developed for open-celled foam-type porous media.

Reference	Unit cell	Expression	Comments
Calmidi and Mahajan[a]		$K = 0.00073 D_p^2 (1-\varepsilon)^{-0.224} (d_f/D_p)^{-1.11}$ where $\frac{d_f}{D_p} = 2\sqrt{\frac{1-\varepsilon}{3\pi}}\frac{1}{G}$ and $G = 1 - e^{-(1-\varepsilon)/0.04}$	– Empirical correlation. – The ratio of ligament thickness to the pore diameter d_f/D_p is empirically fitted considering a shape factor G for various ligament shapes. – Valid ranges of porosity $\varepsilon = 0.9$–0.97 and pore density of 5–40 pores per inch (PPI).
Hooman and Dukhan[b]		$K \sim 0.054 D_p^2 \varepsilon\sqrt{1-\varepsilon}$	– Semi-analytical solution based on the hydraulic resistance network and scale analysis at pore level. – Valid ranges of porosity $\varepsilon = 0.75$–0.97 and pore density of 10–40 PPI.
Bhattacharya et al.[c]		$\frac{K}{d^2} = \frac{\varepsilon^2}{36\chi(\chi-1)}$ where $\frac{1}{\chi} = \frac{\pi}{4\varepsilon}[1 - (1.18\sqrt{\frac{1-\varepsilon}{3\pi}}\frac{1}{G})^2]$ and $G = 1 - e^{-(1-\varepsilon)/0.04}$	– Extended and modified Du Plessis's analytical model. – A modified tortuosity model but physical invalid since it contradicts both the trend and the high porosity limit (void fraction = 1) for most of the experimental results reported in the literature where tortuosity is reduced as the porosity is increased. – With semi-empirical fitting factors G. – Valid ranges of porosity $\varepsilon = 0.9$–0.97 and pore density of 5–40 PPI.
Depois and Mortensen[d]		$K = \frac{D_p^2(1-\varepsilon)}{4\pi}(\frac{\varepsilon-\varepsilon_0}{3\varepsilon})^{3/2}$ where $\varepsilon_0 = 1-\rho^*$, the initial void fraction of the packed spherical particles, $\varepsilon_0 = 0.36$ for random dense packing of monosized spheres	– Analytical solution based on the similarity between the shape of pores of foams and that of sintered spherical particle in a dry powder compact. – The analogy between foams and bed of spherical particles has no physical meaning due to the high void inside the virtual packed bed (void fraction $\varepsilon > 0.9$). However, this analogy allows the direct determination of the estimation of the pressure drops in the open-cell foams in a simple and reliable way but needs further studies. – Valid ranges of porosity $\varepsilon = 0.7$–0.97 and pore density of 5-40 PPI
Dukhan[e]		$K = a\exp(b\varepsilon)$ where $a = 1\times10^{-11}$, $b = 0.1$ for 10 PPI $a = 9\times10^{-12}$, $b = 0.1$ for 20 PPI $a = 8\times10^{-15}$, $b = 0.16$ for 40 PPI	– Empirical correlation – Valid ranges of porosity $\varepsilon = 0.68$–0.92 and pore density of 10–40 PPI.
Tadrist et al.[f]		$K = \frac{\varepsilon^3 d_f^2}{\alpha(1-\varepsilon)^2}$ where $\alpha = 100$–865	– Empirical correlation – Valid ranges of porosity $\varepsilon = 0.885$–0.933 and pore density of 10-40 PPI.

[a] See Ref. [22].
[b] See Ref. [23].
[c] See Ref. [32].
[d] See Ref. [24].
[e] See Ref. [26].
[f] See Ref. [29].



(a) (b)

Fig. 1. High porosity aluminum (Al) foam with open cells ($\varepsilon = 0.92$); (a) Photograph of Al foam; (b) Microstructure of Al foam from scanning electronic microscope (SEM).

attempted by Bhattacharya et al. [33] to a wider porosity range (0.90–0.98), but unfortunately, empirical fitting factors were introduced.

An analytical model for permeability without any fitting factors even if a certain degree of approximation needs to be made, is desirable. In this letter, we present a permeability model that is fully analytical for two-phase open-cell porous media e.g., foams (see Fig. 1). The model is expressed as a function of porosity and pore size and is valid in a wide range of these two parameters.

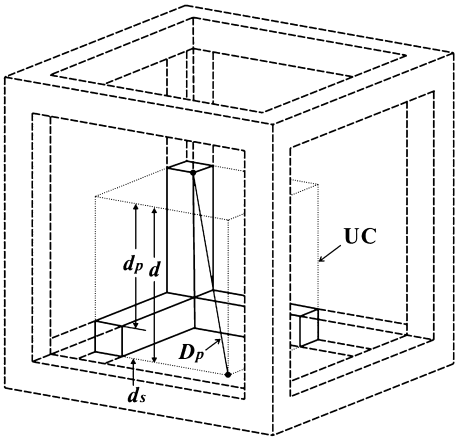


Fig. 2. A representative unit cell (UC) for open-cell foams [31,32].

2. A cubic cell based analytical modelling

A morphological description of porous media plays an essential role in modeling transport phenomena. There exists a representative unit cell that is distributed homogeneously within an isotropic porous medium, which enables a unit cell modeling to be applicable. A cubic unit cell was adopted and used to derive a general permeability model by Du Plessis et al. [31] and Fourie and Du Plessis [32] based on volumetric averaging. A final expression for permeability is given as,

$$\frac{K}{d^2} = \frac{\varepsilon^2}{36\chi(\chi-1)} \tag{3}$$

where d is a microscopic length scale which has relations with a pore size and a cross-sectional pore dimension as [31],

$$D_p = (\sqrt{3})d_p = \sqrt{\frac{3\varepsilon}{\chi}}d \tag{4}$$

Here, χ is the tortuosity of fluid flow across the porous solid matrix. Given the three-dimensional pores in porous media, the diagonal length of the cubic unit cell i.e., D_p in Fig. 2, is more appropriate as a measure of pore size.

Du Plessis et al. [31] proposed this geometrical pore-scale model (Eq. (3)) for predicting pressure drop in a Newtonian fluid flowing through highly porous, isotropic metallic foams. This model was found to be capable of predicting the hydrodynamic conditions in both the Darcy and Forchheimer regimes, without a priori knowledge of the flow behavior of the particular metallic foam, but is applicable in a limited valid range for pore size (45–100PPI) and porosity (0.973–0.978). When applied to wider porosity range, this model significantly overestimated (1.4–2.7 times higher) permeability of open-cell metallic foams in a relative wider porosity range of 0.9–0.98 [33] due to the improper tortuosity, which reflects the extent of the total winding path length available within the representative unit cell of a porous medium for flow to the basic straight stream-wise length scale of the porous structure.

As seen in Eq. (3), permeability is a product of tortuosity, porosity and pore size. Therefore, an accurate description of the tortuous flow path in porous media underlies the physical basis to model transport properties. Adopting the branching algorithm based on particle tracing through the entire branching of highly tortuous flow paths, Beeckman [34] developed a theoretical tortuosity model for porous media with highly interconnected pores for a full porosity range. We adopt the Beeckman theoretical model for tortuosity of open-cell metallic foams due to the topological similarity, expressed as:

$$\chi = \frac{\varepsilon}{1-(1-\varepsilon)^{1/3}} \tag{5}$$

Substitution of (5) into (3) gives a model for permeability as a function of porosity and pore size as:

$$K = \frac{\varepsilon[1-(1-\varepsilon)^{1/3}]}{108[(1-\varepsilon)^{1/3}-(1-\varepsilon)]}D_p^2 \tag{6a}$$

or

$$\frac{K}{d^2} = \frac{\varepsilon[1-(1-\varepsilon)^{1/3}]^2}{36[(1-\varepsilon)^{1/3}-(1-\varepsilon)]} \tag{6b}$$

Once the porosity (ε) and cross-sectional pore size (D_p) are determined, permeability of a given isotropic porous media can be analytically estimated by Eq. (6).

3. Results and discussion

We incorporate a tortuosity model based on the branching algorithm method [34] into a permeability model developed through a cubic unit cell modeling [31,32]. The present analytical model

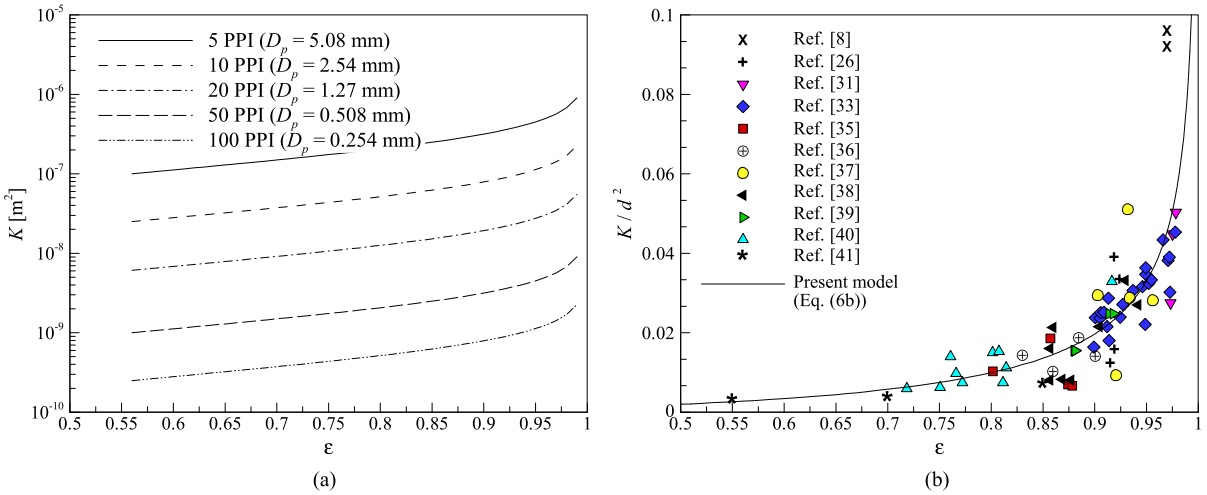


Fig. 3. Dependence of permeability on porosity and pose size of isotropic porous media (foams), having a porosity range of $\varepsilon = 0.55$–0.98 and a pore size range from 0.254 mm to 5.08 mm: (a) Permeability (K) vs. porosity and pore size; (b) Unified dependence of dimensionless permeability (K/d^2) on porosity.

(Eq. (6a)) indicates that permeability depends on porosity and pore size for isotropic porous media, showing that permeability is increased with an increase in porosity but decreased with an increase in pore size (Fig. 3a). It is easier for convective flow to travel in porous media having larger pores for a given porosity, and having higher porosity for a given pore size. It is intuitive that with the porosity increased whilst maintaining pore size, the ligaments constructing the pore structure, are thinned, leading to a reduced flow resistance and tortuosity. As a result, a higher permeability value exists.

Further, the normalization of permeability by pore size can lead to a sole dependence on porosity. Eq. (6b) appears to unify the data sets from experiments and numerical simulations [8,26,31,33,35–41] in a wide range of porosity ($0.55 \leq \varepsilon < 1.0$).

It can be infer that the better description of tortuosity makes the unit cell model capable of predicting permeability as a function of a measurable parameter, porosity. The present analytical model is based on physical principles with no empirical parameters.

4. Conclusions

This study presents an analytical model that can predict permeability for isotropic porous media. Permeability is analytically expressed as a function of porosity and pore size, which provides how permeability is varied with these two morphological parameters. Further, permeability can be scaled with a characteristics length (d) which is a function of actual pore size and porosity: its dimensionless form, K/d^2 depends only on porosity.

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