

# Effects of Interface Bonding on Acoustic Wave Generation in an Elastic Body by Surface-Mounted Piezoelectric Transducers

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**Abstract**—We study the effects of interface bonding on acoustic wave generation in an elastic body using surface-mounted piezoelectric transducers driven electrically. A theoretical analysis is performed based on a physical model of a piezoelectric layer on an elastic substrate. The transducer–substrate interface is described by the shear-slip model, representing a viscoelastic interface. Different from the results in the literature on free vibrations of structures with weak interfaces, this paper presents an electrically forced vibration analysis. An analytical solution for the generated acoustic wave is obtained and used to calculate its energy flux and the efficiency of the transduction. The effects of the interface parameters are examined. It is found that the interface bonding affects the performance of the transducer in multiple ways, some of which may be exploitable in designs for better transducer performance. In particular, optimal transduction is not necessarily associated with a perfectly bonded interface.

## I. INTRODUCTION

MULTILAYERED plates with interfaces among layers of different materials are common structures in many engineering fields. Very often a gluing substance, e.g., epoxy, is applied to the interfaces. Alternatively, the molecules of the neighboring materials may interpenetrate into each other to form a bonding layer. Ideally, in the simplest mechanics description, an interface is treated as a geometric surface where perfect or rigid bonding with continuous displacement and traction is assumed. In reality, an interface may be viewed physically as a thin layer with its own material properties. People have developed interface models with different levels of sophistication. Interface properties and their effects have been studied experimentally [1]–[4] and theoretically for both static and dynamic problems [5]–[8]. Specifically, for time-harmonic motions relevant to the present paper, wave propagation in unbounded do-

mains [9]–[21] and vibrations of finite bodies [22]–[26] have been analyzed for various applications including material characterization, structural strength consideration, acoustic wave sensors, and nondestructive evaluation. These are mostly free vibration analyses regarding wave speed or frequency. More references can be found in a review article [27]. Examination of the literature shows the lack of electrically forced vibration analyses of transducers with weak interfaces. A forced vibration analysis is necessary to obtain transducer admittance and efficiency, which are fundamental transducer characteristics.

The present paper is concerned with the effects of interface bonding on acoustic wave generation in an elastic body with surface-bonded piezoelectric transducers driven electrically. In conventional analyses of this type of problem, perfect bonding between the piezoelectric transducer and the elastic body is routinely assumed. The effects of the viscoelasticity of epoxy or other glues at an interface on the electrical admittance of the transduction structure, the energy flux of the acoustic wave generated, and the efficiency of the transduction have not been well studied and understood. Our theoretical analysis is based on a mechanics model of a piezoelectric layer as a transducer bonded to an elastic substrate with their interface described by the shear-slip model [6], [7] for an imperfect, weak, or nonrigid interface. This interface model allows for a discontinuity of the tangential displacements across the interface while the traction is still continuous. The interface has its own viscoelasticity but not inertia. Such a model can describe the most basic behavior of an interface and has often been used in free vibration analyses. The shear-slip model will be introduced in Section II, along with a piezoelectric–elastic structure and its governing equations. An analytical solution is obtained in Section III. Numerical results and discussions are presented in Section IV. Some conclusions are drawn in Section V.

## II. GOVERNING EQUATIONS

Consider the structure of a piezoelectric layer on an elastic half-space, shown in Fig. 1. The piezoelectric layer is ceramic poled along the  $x_3$ -direction, which is determined from  $x_1$  and  $x_2$  by the right-hand rule. It is electroded at its two surfaces and is driven by an alternating voltage across the electrodes. Because of the specific orientation of the poling direction of the ceramics and the direction of the applied electric field, shear-horizontal or

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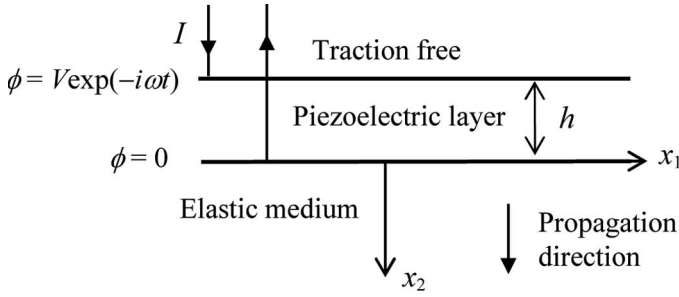


Fig. 1. A ceramic plate on an elastic half space.

antiplane waves are generated in the elastic half-space, propagating away from the ceramic plate.

For the purpose of this paper, it is sufficient to consider motions independent of  $x_1$  and  $x_3$ . The only mechanical displacement component  $u_3 = u$  and the electrical potential  $\phi$  can be written as

$$u = u(x_2, t), \quad \phi = \phi(x_2, t). \quad (1)$$

They are governed by [28]–[30]

$$\begin{aligned} \bar{c}\nabla^2 u &= \rho\ddot{u}, & \nabla^2 \psi &= 0, & \phi &= \psi + \frac{e}{\varepsilon}u, & -h < x_2 < 0, \\ \hat{c}\nabla^2 u &= \hat{\rho}\ddot{u}, & & & & & x_2 > 0, \end{aligned} \quad (2)$$

where  $\psi$  is a function introduced for convenience [28]–[30].  $\bar{c} = c_{44} + e_{15}^2/\varepsilon_{11}$  and  $\rho$  are the piezoelectrically stiffened shear elastic constant and the mass density of the ceramic layer.  $\varepsilon = \varepsilon_{11}$  and  $e = e_{15}$  are the dielectric and piezoelectric constants of the layer.  $\hat{c}$  and  $\hat{\rho}$  are the shear elastic constant and mass density of the elastic substrate.  $\nabla^2 = \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2$  is the two-dimensional Laplacian operator. The boundary and continuity conditions at the top of the piezoelectric layer and the interface between the layer and the substrate are

$$T_{23}(x_2 = -h) = 0, \quad (3a)$$

$$\phi(x_2 = -h) - \phi(x_2 = 0) = V \exp(-i\omega t), \quad (3b)$$

$$T_{23}(x_2 = 0^-) = K[u(x_2 = 0^+) - u(x_2 = 0^-)], \quad (3c)$$

$$T_{23}(x_2 = 0^+) = K[u(x_2 = 0^+) - u(x_2 = 0^-)], \quad (3d)$$

where (3c) and (3d) are based on the shear-slip model for an elastic interface.  $K$  is the effective interface elastic constant.  $K = \infty$  represents a perfectly or rigidly bonded interface. When  $K = 0$ , the layer and the substrate lose their mechanical interaction. In harmonic motions, a complex  $K$  describes a viscoelastic interface. In addition to (3), when  $x_2 \rightarrow +\infty$ , the waves generated by the piezoelectric layer must be outgoing toward  $+\infty$  (radiation condition).

The free charge density on the upper electrode at  $x_2 = -h$  and the density of the current flowing into this electrode are given by

$$q = D_2, \quad I = \dot{q} = -i\omega q. \quad (4)$$

Then, the frequency-dependent admittance  $A$  of the structure per unit electrode area can be determined from

$$I = AV. \quad (5)$$

With the complex notation for harmonic motions, the input electrical power is given by

$$P_1 = \frac{1}{4}(IV^* + I^*V), \quad (6)$$

where an asterisk represents a complex conjugate. The instantaneous energy flux of the waves generated in the elastic medium can be calculated from

$$s_2 = -T_{23}\dot{u}, \quad (7)$$

in which real fields must be used, which are taken from the real or imaginary parts of complex fields. We denote its average over a period by  $P_2 = -(T_{23}\dot{u}^* + T_{23}^*\dot{u})/4$ . Then, the efficiency for the acoustic wave generation by the applied electric field is calculated from

$$\eta = P_2/P_1. \quad (8)$$

### III. FORCED VIBRATION SOLUTION

For  $-h < x_2 < 0$ , the solution can be written as

$$\begin{aligned} u &= (A_1 \cos \xi x_2 + A_2 \sin \xi x_2) \exp(-i\omega t), \\ \psi &= (B_1 x_2 + B_2) \exp(-i\omega t), \end{aligned} \quad (9)$$

where  $A_1$ ,  $A_2$ ,  $B_1$ , and  $B_2$  are undetermined constants, and

$$\xi^2 = \frac{\rho\omega^2}{\bar{c}}. \quad (10)$$

The relevant stress component and the electric potential needed for the boundary conditions can be obtained from (9) as

$$\begin{aligned} T_{23} &= \bar{c}u_{,2} + e\psi_{,2} \\ &= (-\bar{c}A_1\xi \sin \xi x_2 + \bar{c}A_2\xi \cos \xi x_2 + eB_1) \exp(-i\omega t), \end{aligned}$$

$$\begin{aligned} \phi &= \psi + \frac{e}{\varepsilon}u \\ &= \left( \frac{e}{\varepsilon}A_1 \cos \xi x_2 + \frac{e}{\varepsilon}A_2 \sin \xi x_2 + B_1 x_2 + B_2 \right) \exp(-i\omega t). \end{aligned} \quad (11)$$

For  $x_2 > 0$ , the solution can be written as

$$u = \hat{A} \exp i(\hat{\xi}x_2 - \omega t), \quad (12)$$

where  $\hat{A}$  is an undetermined constant, and

$$\hat{\xi}^2 = \frac{\hat{\rho}\omega^2}{\hat{c}}. \quad (13)$$

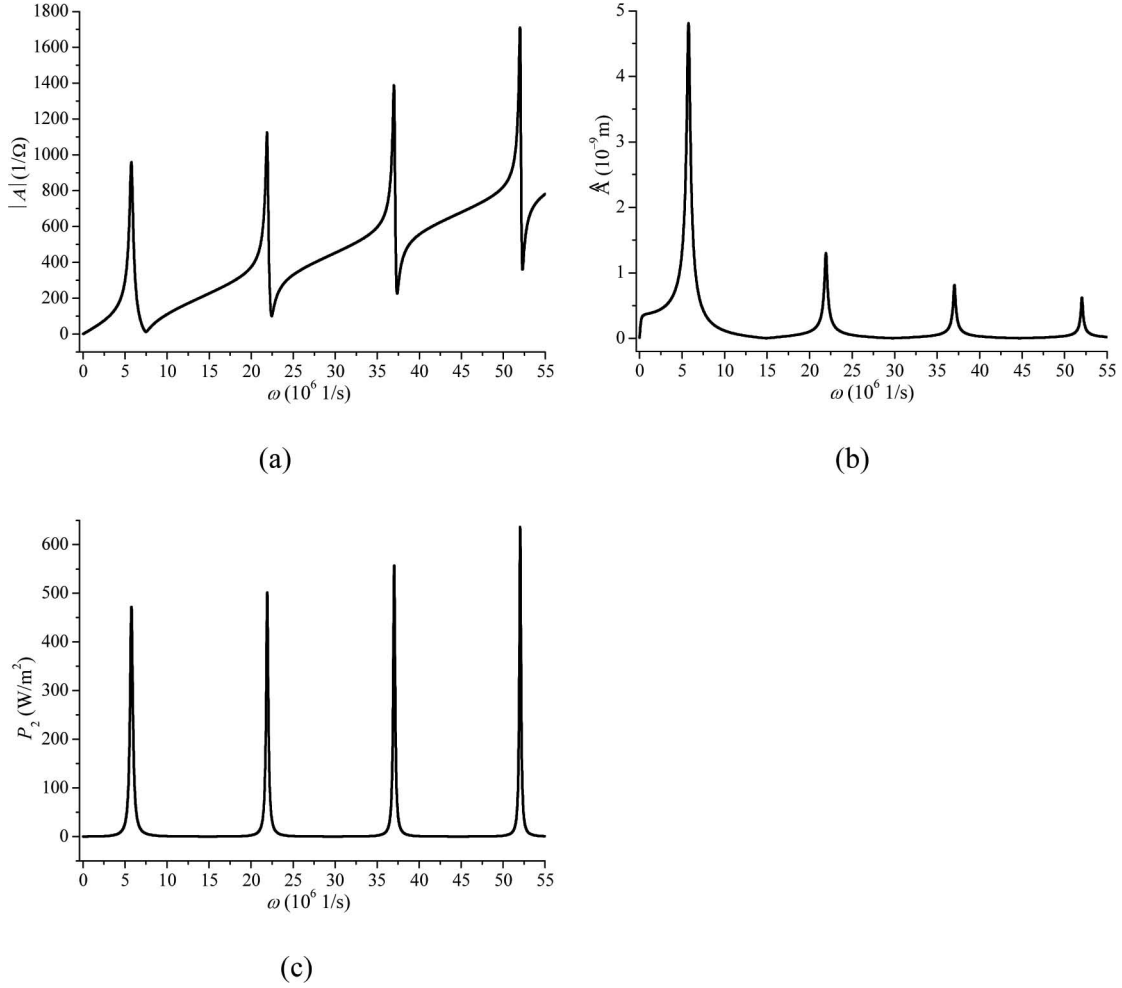


Fig. 2. The first few resonances of (a) admittance  $|A|$ ; (b) amplitude  $|\hat{A}|$  of the generated wave; and (c) energy flux  $P_2$  of the generated wave ( $K_0 = 1 \times 10^{14}$  N/m<sup>3</sup> and  $K_1 = 0.1K_0$ ).

We note that (12) represents waves propagating in the positive direction of the  $x_2$ -axis, which already satisfies the radiation condition. For interface continuity conditions, we need

$$T_{23} = \hat{c}u_{,2} = \hat{c}i\hat{\xi}\hat{A} \exp i(\hat{\xi}x_2 - \omega t). \quad (14)$$

Substituting (9), (11), (12), and (14) into (3) results in the following four linear algebraic equations for  $A_1$ ,  $A_2$ ,  $B_1$ , and  $\hat{A}$ , driven by  $V$ :

$$\begin{aligned} \bar{c}A_1\xi \sin \xi h + \bar{c}A_2\xi \cos \xi h + eB_1 &= 0, \\ \frac{e}{\varepsilon}A_1 \cos \xi h - \frac{e}{\varepsilon}A_2 \sin \xi h - B_1 h + B_2 - \frac{e}{\varepsilon}A_1 - B_2 &= V, \\ \bar{c}A_2\xi + eB_1 &= K(\hat{A} - A_1), \\ \hat{c}i\hat{\xi}\hat{A} &= K(\hat{A} - A_1). \end{aligned} \quad (15)$$

Solving (15), we obtain the amplitude of the generated wave in the substrate as

$$\hat{A} = \frac{e}{\bar{c}} \frac{1 - \cos \xi h}{\Delta} V = \frac{e}{\bar{c}} \frac{1 - \cos \xi h}{\Delta_1 + \Delta_2} V, \quad (16)$$

where

$$\begin{aligned} \Delta_1 &= (1 - \cos \xi h) \left[ k^2(\cos \xi h - 1) - i \frac{\hat{c}}{\bar{c}} \hat{\xi} h \right] \\ &\quad + \left( \sin \xi h + i \frac{\hat{c}\hat{\xi}}{\bar{c}} \right) (\xi h - k^2 \sin \xi h), \\ \Delta_2 &= \frac{i\hat{c}\hat{\xi}}{K} [k^2(2 - 2 \cos \xi h) - \xi h \sin \xi h], \end{aligned} \quad (17)$$

in which  $k^2 = e^2/(\varepsilon\bar{c})$  is the shear electromechanical coupling coefficient of the piezoelectric material.  $\Delta = 0$  yields the frequency equation for free vibration.

#### IV. NUMERICAL RESULTS AND DISCUSSION

As a numerical example, consider a ceramic plate of PZT-5H with  $c = 2.3 \times 10^{10}$  N/m<sup>2</sup>,  $e = 17$  C/m<sup>2</sup>,  $\varepsilon =$

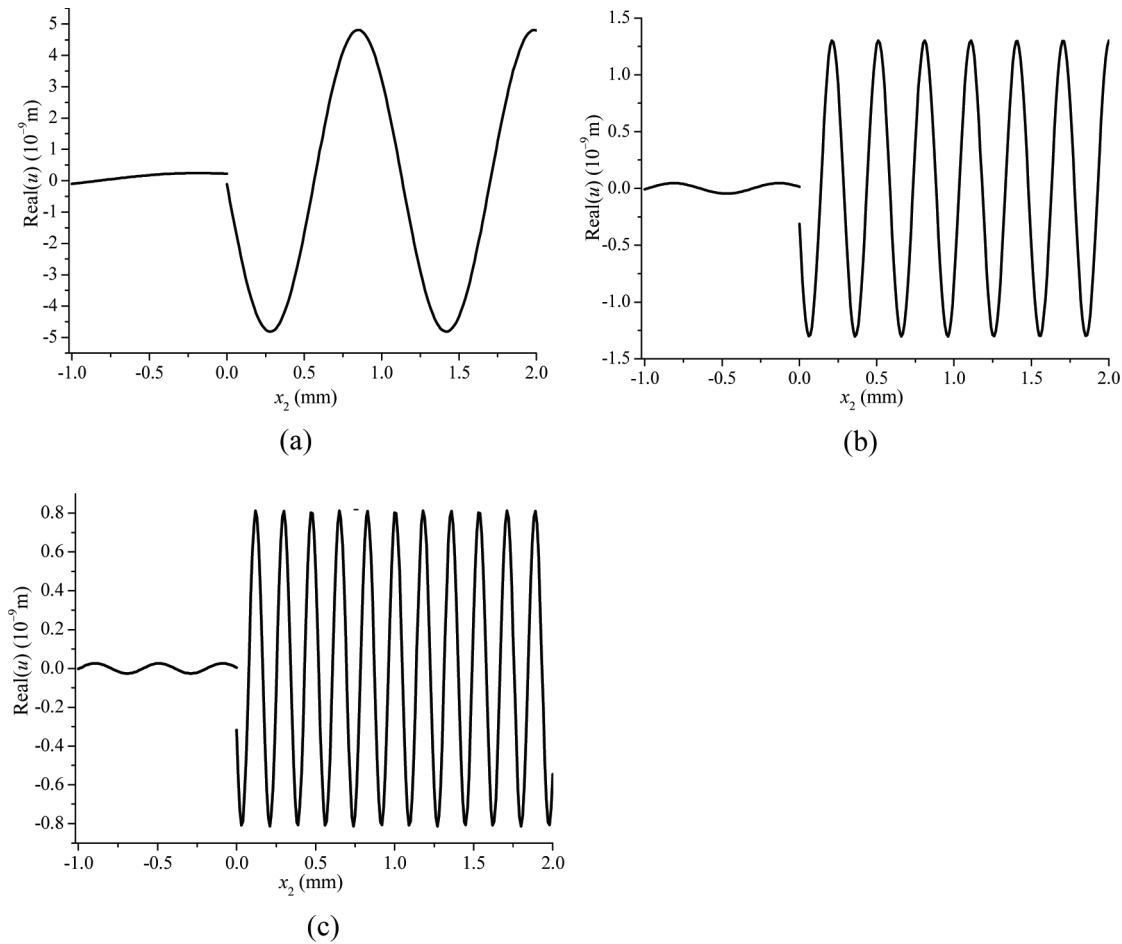


Fig. 3. Displacement distribution along  $x_2$  at the first three resonances.

$1.506 \times 10^{-8}$  F/m, and  $\rho = 7500$  kg/m<sup>3</sup>. For the substrate, we consider polythene [31] with  $\hat{c} = 0.128 \times 10^{10}$  N/m<sup>2</sup> and  $\hat{\rho} = 1180$  kg/m<sup>3</sup>.  $h = 1$  mm and  $V = 1$  V are used. In this case, the substrate is more compliant and lighter than the transducer material. Because the interface glue is usually viscoelastic, we use a complex interface stiffness whose imaginary part describes interface damping. Let  $K = K_0 - iK_1$ , where both  $K_0$  and  $K_1$  are real and positive. A negative imaginary part is used to be consistent with the  $\exp(-i\omega t)$  factor in our complex notation. We begin with  $K_0 = 1 \times 10^{14}$  N/m<sup>3</sup> and  $K_1 = 0.1K_0$ , which will be varied later.  $K_1 = 0.1K_0$  represents relatively large damping, which is chosen to more clearly show the resonances graphically.

Figs. 2(a), 2(b), and 2(c) show the admittance  $|A|$  per unit area, the amplitude  $|\hat{A}|$  of the generated wave, and the energy flux  $P_2$  of the generated wave versus the driving frequency, respectively. The frequencies at which the admittance assumes maxima are the resonant frequencies. At resonances, the amplitude and the output energy flux also assume maxima. At these frequencies, significant acoustic wave generation can be realized.

When  $K_0 = 1 \times 10^{14}$  N/m<sup>3</sup> and  $K_1 = 0.1K_0$ , corresponding to the first three resonances in Fig. 2, the distribution of the displacement  $u$  along  $x_2$  is shown in Fig. 3.

At the interface between the plate and the substrate, the displacement is discontinuous, as described by the shear-slip model. This discontinuity is larger for higher-order modes. For higher-order modes, there are more nodal points (zeros) along the plate thickness. The wavelength becomes shorter in both the plate and the substrate for higher-order modes. In the example shown, the amplitude in the substrate is significantly larger than that in the plate, whereas the energy flux in Fig. 2(c) does not change much for different modes.

The effects of the interface stiffness  $K_0$  on the admittance  $|A|$ , the amplitude  $|\hat{A}|$  of the generated wave, and the energy flux  $P_2$  of the generated wave near the first resonance locally are shown in Fig. 4, where  $K_1 = 2 \times 10^{13}$  N/m<sup>3</sup>. The resonant frequency is sensitive to the interface stiffness, as expected. When  $K_0$  decreases, the resonant frequency decreases because there is less stiffness in the structure. It is interesting and potentially useful to see that when the interface bonding becomes weaker, the peak values of  $|A|$ ,  $|\hat{A}|$ , and  $P_2$  increase slightly, showing vibrations with larger amplitude in the substrate and stronger acoustic wave generation.

Fig. 5 presents a global view of the effects of  $K_0$  on the admittance  $|A|$ , the amplitude  $|\hat{A}|$ , the energy flux  $P_2$ , and the efficiency  $\eta$  when  $K_1 = 0.1K_0$ . Cases 1 and 2 in the

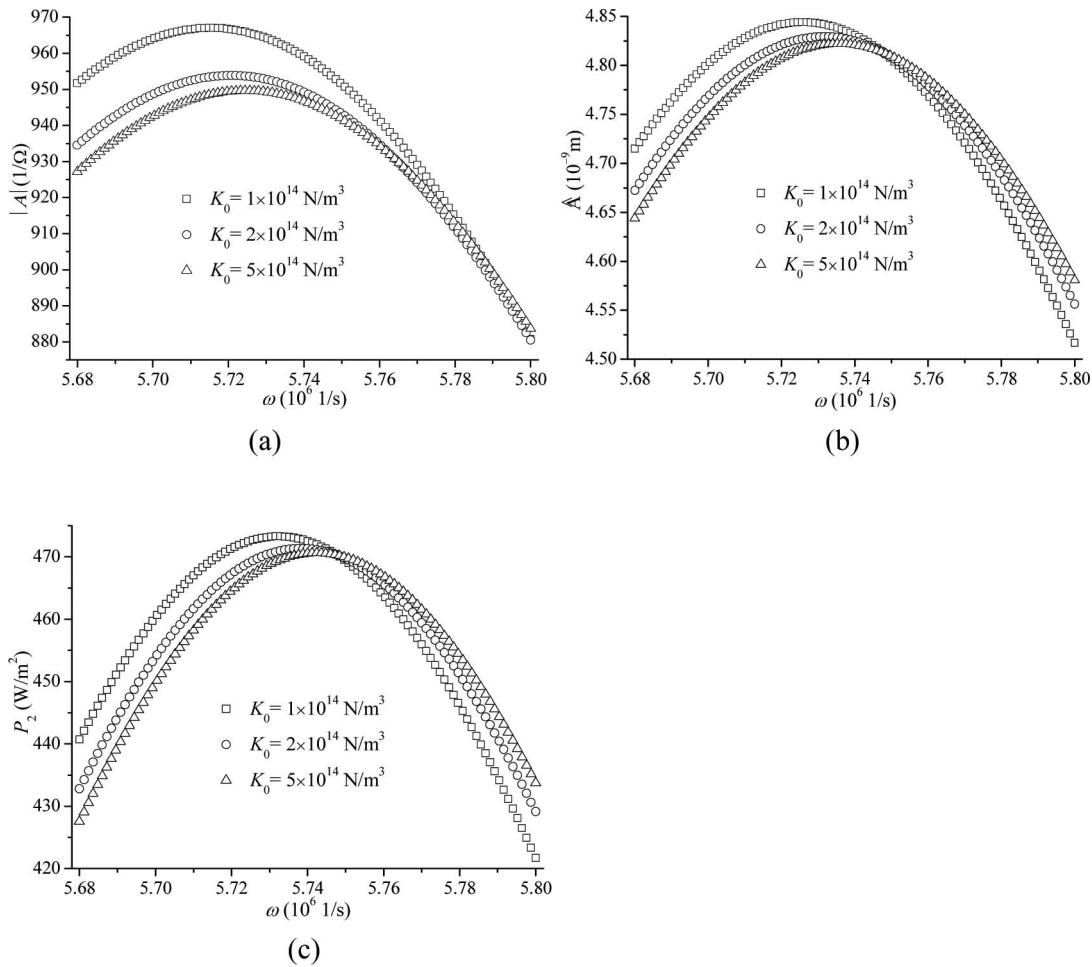


Fig. 4. (a) Admittance  $|A|$ , (b) amplitude  $|\hat{A}|$  of the generated wave, and (c) energy flux  $P_2$  of the generated wave near the first resonance for different  $K_0$  ( $K_1 = 2 \times 10^{13}$  N/m<sup>3</sup>).

figure are for two different driving frequencies. In Case 1, the driving frequency follows the first resonant frequency as  $K_0$  changes. In Case 2, the driving frequency is fixed to be the first resonant frequency when  $K_0 = 1 \times 10^{14}$  N/m<sup>3</sup>, which is  $\omega = 5.725156 \times 10^6$  1/s. The figure shows that for large  $K_0$ , the corresponding  $|A|$ ,  $|\hat{A}|$ ,  $P_2$ , and  $\eta$  approach their limit values when the interface is perfectly bonded. When  $K_0$  is smaller or when the interface becomes weaker,  $|A|$ ,  $|\hat{A}|$ , and  $P_2$  all increase, but the efficiency  $\eta$  drops monotonically. For  $|A|$ ,  $|\hat{A}|$ , and  $P_2$ , there exist maxima at the same value of  $K_0$ . At this  $K_0$ , the ef-

iciency is slightly less than its maximum for large  $K_0$  or a perfectly bonded interface. Therefore, this particular value of  $K_0$  at which  $|A|$ ,  $|\hat{A}|$ , and  $P_2$  assume maxima with a reasonably high efficiency may be viewed as the optimal interface, which is not a perfectly bonded interface. Hence, a weak interface is not necessarily unfavorable and should be considered in the overall design of the transduction structure for optimal performance.

The interface is viscoelastic, as described by a complex  $K$ . Table I shows the effect of the interface damping represented by  $K_1$ , i.e., the imaginary part of  $K$ . From the

TABLE I. EFFECT OF INTERFACE DAMPING DESCRIBED BY  $K_1$ .

$K_0$ (N/m <sup>3</sup> )	$K_1/K_0$	$\omega$ (10 <sup>6</sup> 1/s)	$ \hat{A} $ (10 <sup>-9</sup> m)	$ A $ (1/ $\Omega$ )	$P_2$ (W/m <sup>2</sup> )	$\eta$
$1 \times 10^{14}$	0	5.724721	4.846585	953.46	473.04	1.0
	0.01	5.724754	4.846534	954.12	473.03	0.9993
	0.1	5.725165	4.845740	959.82	472.95	0.9931
$2 \times 10^{14}$	0	5.731473	4.829666	949.15	470.85	1.0
	0.01	5.731485	4.829653	949.47	470.85	0.9996
	0.1	5.731653	4.829415	952.37	470.83	0.9965
$5 \times 10^{14}$	0	5.735595	4.822336	947.66	470.10	1.0
	0.01	5.735598	4.822333	947.80	470.10	0.9999
	0.1	5.735656	4.822270	948.96	470.09	0.9986

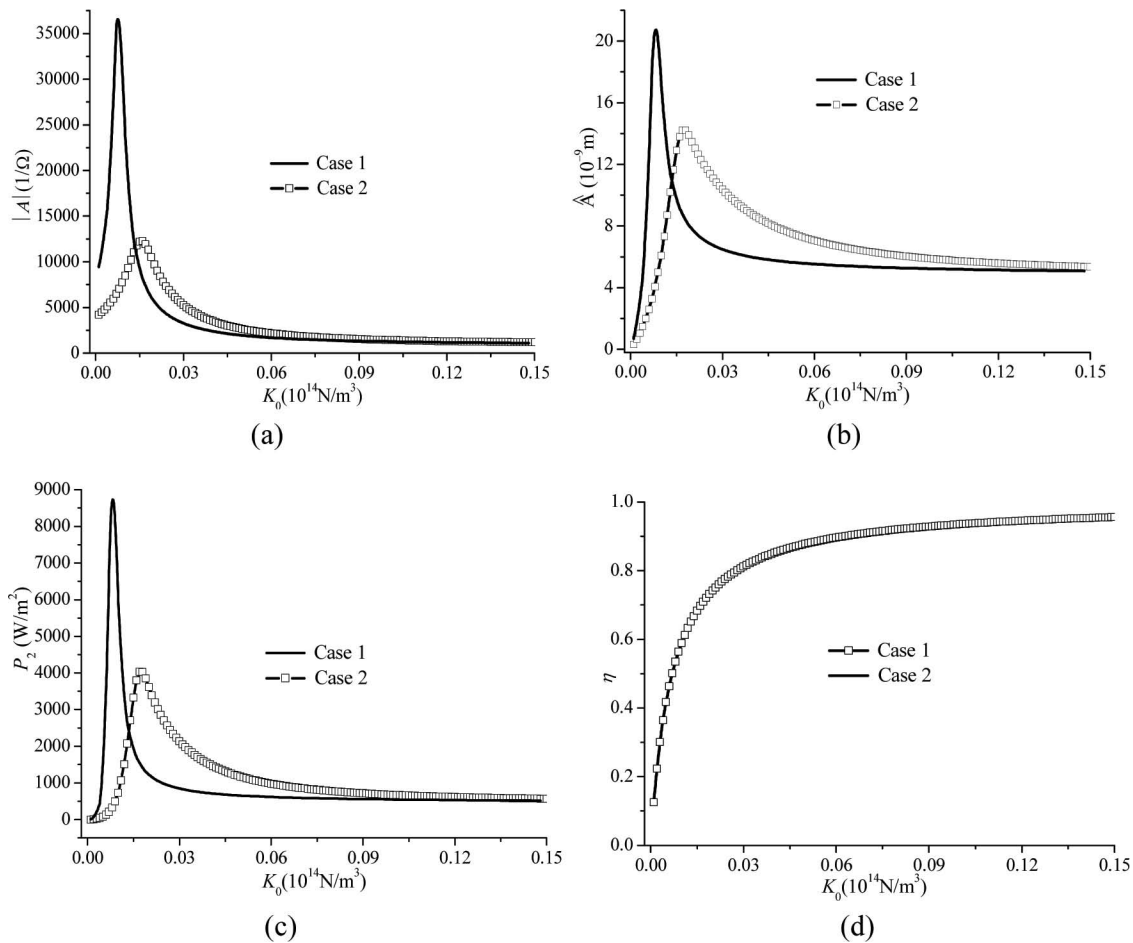


Fig. 5. (a) Admittance  $|A|$ , (b) amplitude  $|\hat{A}|$  of the generated wave, and (c) energy flux  $P_2$  of the generated wave, and (d) efficiency  $\eta$  versus  $K_0$  ( $K_1 = 0.1K_0$ ).

table, it is seen that overall, the effect of  $K_1$  is small. This is because the acoustic wave generated carries the major part of the energy flow which serves as the radiation damping of the transducer and dominates the interface damping.

## V. CONCLUSION

The shear-slip interface model used leads to a jump discontinuity of the tangential displacement at the interface. The interface bonding as part of the stiffness of the whole structure affects the overall transducer performance including the resonant frequency, the amplitude, and energy flux of the acoustic wave generated, the admittance of the structure, and the efficiency of electric-acoustic energy conversion. When the interface bonding is imperfect, although the efficiency drops monotonically, the amplitude and energy flux of the acoustic wave generated, as well as the admittance of the structure, may increase and assume maxima for a particular value of the interface stiffness. The interface damping is a relatively small effect compared with the radiation damping of the acoustic wave

generated. Interface analysis should be part of the transduction design for optimal performance. Although the analysis is for shear waves, with a different interpretation of the notation, the analysis is also valid for longitudinal waves generated by a ceramic layer with thickness poling and an interface that is elastic in the normal direction. Therefore, similar behaviors are expected for longitudinal wave generation by piezoelectric transducers.

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