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Propagation of the Bleustein–Gulyaev waves in a functionally graded transversely isotropic electro-magneto-elastic half-space

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A B S T R A C T

Bleustein–Gulyaev (B–G) waves in a functionally graded transversely isotropic electro-magneto-elastic half-space, in which all parameters exponentially change along the depth direction, are investigated, and the dispersion equations with respect to the phase velocity for electromagnetically open and shorted conditions are obtained. The B–G waves are dispersive because of the inhomogeneity of the material, which is different from a pure substrate. The graded coefficient exhibits different effects on the B–G waves at different electromagnetic circumstances. With the intervention of the functionally graded material, the electro-magneto-mechanical coupling factor can be improved and the penetration depth of the B–G waves can be decreased. The results can be used in the design of high-performance surface acoustic wave devices.

1. Introduction

Surface waves have been successfully applied in many technological fields, such as the non-destructive evaluation (NDE) of materials, resonators, filters, and sensors, among others (Yang and Wang, 2008). Nearly 40 years ago, Bleustein (1968) and Gulyaev (1969) simultaneously discovered a kind of shear horizontal electro-acoustic surface wave that can propagate in a type of transversely isotropic piezoelectric media; this wave is currently known as the Bleustein–Gulyaev (B–G) wave. The B–G surface wave is a unique result in the repertoire of the surface acoustic wave (SAW) theory, which has no counterpart in elastic materials.

Since then, the B–G wave theory has become one of the cornerstones of modem electro-acoustic technology and many researchers have thoroughly investigated the propagation characteristics of the B–G wave and its applications in SAW devices (Bleustein, 1969; Li, 1996), including the effects of the initial stress (Liu et al., 2003), the fully electromagnetic coupling (Yang, 2000), the piezoelectric layered structure (Jin et al., 2001), and the imperfect interface influence (Liu et al., 2010a), among others. In addition, because the B–G wave does not radiate energy into the contacting liquid and is sensitive to the changes in the liquid density and the viscous coefficient, it can be a promising candidate for liquid sensing applications (Zhang et al., 2001). For example, Kieiczyhskai and Plowiec (1989) presented a method of measuring the rheological properties of viscoelastic liquids via the perturbation theory using B–G waves. Guo and Sun, 2008 investigated the B–G wave propagation in half-space piezoelectric materials loaded with viscous liquids.

The appearance of magneto-electro-elastic media, which simultaneously exhibit piezoelectric, piezomagnetic, and magneto-electric properties, offers great opportunities for the creation of intelligent structures and devices, because this kind of material possesses the capacity to respond to internal and/or environmental changes. The wave properties in magneto-electro-elastic materials, such as the propagation of harmonic waves in homogeneous (Chen et al., 2007) and functionally graded magneto-electro-elastic multilayered plates (Chen et al., 2005), the shear horizontal waves (Wang et al., 2007; Soh and Liu, 2006) and the Lamb waves in the piezoelectric–piezomagnetic bimaterial (Wu et al., 2007), the Love waves in the 2–2 piezoelectric/piezomagnetic structures (Liu et al., 2010b), and so on, have been widely investigated.

In the current study, the properties of B–G waves in a functionally graded transversely isotropic magneto-electro-elastic half-space are investigated. The structure of the magneto-electro-elastic substrate and the corresponding mathematical problem are

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defined in Section 2. The analytical solutions of the phase velocity for the electromagnetically open and shorted conditions are then obtained. The numerical calculations based on the analytical solutions are presented in Section 4. Finally, some conclusions are drawn in Section 5.

2. Statement of the problem

Consider an inhomogeneous transversely isotropic electro-magneto-elastic half-space, which is poled in the z-direction following the right-hand rule from the x- and y-axes (Fig. 1). The material parameters in the substrate gradually change along the x-axis.

The B–G waves in such a functionally graded electro-magneto-elastic half-space are to be taken into account. Without loss of any generality, the wave propagation in the positive direction of the y-axis may be written in the following forms:

$$\begin{aligned} u &= v = 0, \quad w = w(x,y,t), \quad 0 \le x < +\infty \\ \varphi &= \varphi(x,y,t), \quad \psi = \psi(x,y,t), \quad -\infty < x < +\infty \end{aligned} \tag{1}$$

where u, v, and w are the mechanical displacement components, φ is the electrical potential function, and ψ is the magnetic potential function representing the motion.

The equilibrium equations of elasticity without the body forces and Gauss' law of electromagnetic statics without a free charge and current are given as follows (Yang, 2006):

$$\sigma_{ij,j} = \rho \ddot{u}_i, \quad D_{i,i} = 0, \quad B_{i,i} = 0 \tag{2}$$

and the constitutive relations (Benveniste, 1995) are

$$\begin{aligned} \sigma_{ij} &= c_{ijkl}u_{k,l} + e_{lij}\varphi_{,l} + q_{kij}\psi_{,k} \\ D_i &= e_{ikl}u_{k,l} - \varepsilon_{il}\varphi_{,l} - \alpha_{ik}\psi_{,k} \\ B_i &= q_{ikl}u_{k,l} - \alpha_{il}\varphi_{,l} - \mu_{ik}\psi_{,k} \end{aligned} \tag{3}$$

where σ_{ij} is the stress tensor, D_i is the electric displacement field, B_i is the magnetic induction, ρ is the mass density of the electro-magneto-elastic material, and c_{ijkl}, e_{kij}, q_{kij}, ε_{il}, α_{ik}, and μ_{ik} are the elastic, piezoelectric, piezomagnetic, dielectric, electromagnetic tensor, and permeability tensor constants, respectively. The dot denotes the time differentiation, the comma followed by the subscript indicates the space coordinate differentiation with respect to the corresponding coordinate, and the repeated subscript index implies a summation with respect to itself. From Eqs. (1)–(3), the coupled field equations for the functionally graded transversely isotropic electro-magneto-elastic half-space are given by

$$\begin{aligned} & c_{44}(x)\nabla^2 w + \frac{\partial c_{44}(x)}{\partial x}\frac{\partial w}{\partial x} + e_{15}(x)\nabla^2\varphi + \frac{\partial e_{15}(x)}{\partial x}\frac{\partial \varphi}{\partial x} + q_{15}(x)\nabla^2\psi + \frac{\partial q_{15}(x)}{\partial x}\frac{\partial \psi}{\partial x} = \rho(x)\frac{\partial^2 w}{\partial t^2} \\ & e_{15}(x)\nabla^2 w + \frac{\partial e_{15}(x)}{\partial x}\frac{\partial w}{\partial x} - \varepsilon_{11}(x)\nabla^2\varphi - \frac{\partial \varepsilon_{11}(x)}{\partial x}\frac{\partial \varphi}{\partial x} - \alpha_{11}(x)\nabla^2\psi - \frac{\partial \alpha_{11}(x)}{\partial x}\frac{\partial \psi}{\partial x} = 0 \\ & q_{15}(x)\nabla^2 w + \frac{\partial q_{15}(x)}{\partial x}\frac{\partial w}{\partial x} - \alpha_{11}(x)\nabla^2\varphi - \frac{\partial \alpha_{11}(x)}{\partial x}\frac{\partial \varphi}{\partial x} - \mu_{11}(x)\nabla^2\psi - \frac{\partial \mu_{11}(x)}{\partial x}\frac{\partial \psi}{\partial x} = 0 \end{aligned} \tag{4}$$

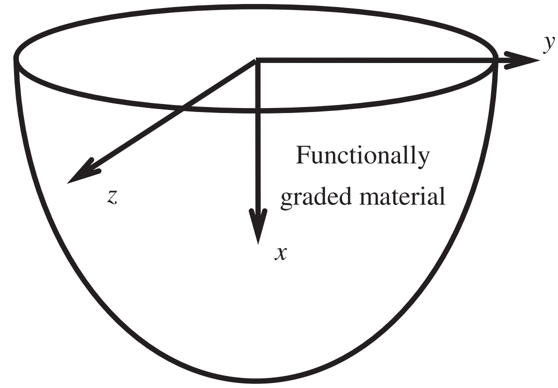

Fig. 1. Geometry of the functionally graded half-space and its coordinate system.

and the nonzero stress, electric displacement, and magnetic induction components needed for the boundary conditions are

$$\begin{aligned} \sigma_{zx} &= \partial c_{44}(x)\frac{\partial w}{\partial x} + e_{15}(x)\frac{\partial \varphi}{\partial x} + q_{15}(x)\frac{\partial \psi}{\partial x} \\ D_x &= e_{15}(x)\frac{\partial w}{\partial x} - \varepsilon_{11}(x)\frac{\partial \varphi}{\partial x} - \alpha_{11}(x)\frac{\partial \psi}{\partial x} \\ B_x &= q_{15}(x)\frac{\partial w}{\partial x} - \alpha_{11}(x)\frac{\partial \varphi}{\partial x} - \mu_{11}(x)\frac{\partial \psi}{\partial x} \end{aligned} \tag{5}$$

where $\nabla^2 = \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2$ is the Laplace operator. In the current study, we assume that all material parameters in the functionally graded electro-magneto-elastic substrate have the same exponential function variation along the x-axis (Qian et al., 2009; Du et al., 2007), which are expressed as follows:

$$\begin{aligned} & c_{44}(x) = c_{44}^0 e^{2\beta x}, \quad e_{15}(x) = e_{15}^0 e^{2\beta x}, \quad q_{15}(x) = q_{15}^0 e^{2\beta x}, \\ & \rho(x) = \rho^0 e^{2\beta x}, \varepsilon_{11}(x) = \varepsilon_{11}^0 e^{2\beta x}, \quad \alpha_{11}(x) = \alpha_{11}^0 e^{2\beta x}, \\ & \mu_{11}(x) = \mu_{11}^0 e^{2\beta x} \end{aligned} \tag{6}$$

where β is the negative exponential coefficient indicating the profile of the material gradient along the x-axis (Qian et al., 2008), and the quantities with superscript 0 are the corresponding values of these parameters at the free surface. Even though these conditions are somewhat difficult to achieve in practice, investigating and understanding the effects of the functionally graded materials on the characteristics of the B–G waves would still be helpful.

Substituting Eq. (6) into Eq. (4) yields

$$\begin{aligned}
&c_{44}^0\left(\nabla^2 w+2\beta\frac{\partial w}{\partial x}\right)+e_{15}^0\left(\nabla^2\varphi+2\beta\frac{\partial\varphi}{\partial x}\right)\\
&\quad+q_{15}^0\left(\nabla^2\psi+2\beta\frac{\partial\psi}{\partial x}\right)=\rho^0\frac{\partial^2 w}{\partial t^2}\\
&e_{15}^0\left(\nabla^2 w+2\beta\frac{\partial w}{\partial x}\right)-\varepsilon_{11}^0\left(\nabla^2\varphi+2\beta\frac{\partial\varphi}{\partial x}\right)-\alpha_{11}^0\left(\nabla^2\psi+2\beta\frac{\partial\psi}{\partial x}\right)=0\\
&q_{15}^0\left(\nabla^2 w+2\beta\frac{\partial w}{\partial x}\right)-\alpha_{11}^0\left(\nabla^2\varphi+2\beta\frac{\partial\varphi}{\partial x}\right)-\mu_{11}^0\left(\nabla^2\psi+2\beta\frac{\partial\psi}{\partial x}\right)=0
\end{aligned}\tag{7}$$

Usually, the space above the upper surface of the layer is air. Let $\varphi_0(x,y,t)$ and $\psi_0 = \psi(x,y,t)$ be the electrical potential and the magnetic potential functions in air ($x \le 0$), respectively. Therefore, $\varphi_0(x,y,t)$ and $\psi_0 = \psi(x,y,t)$ satisfy the Laplace equation, which is expressed as

$$\nabla^2\varphi_0 = 0,\ \nabla^2\psi_0 = 0 \tag{8}$$

Similarly, the electrical displacement and magnetic induction components in air are

$$D_{x0} = -\varepsilon_0\frac{\partial\varphi_0}{\partial x},\ B_{x0} = -\mu_0\frac{\partial\psi_0}{\partial x} \tag{9}$$

where ε_0 and μ_0 are the dielectric and permeability constants in air, respectively.

The traction-free surface at $x = 0$ is given by

$$\sigma_{zx}(0,y) = 0 \tag{10}$$

The electrical and magnetic boundary conditions at $x = 0$ for the electromagnetically open case are

$$\begin{aligned}
\varphi(0,y) &= \varphi_0(0,y),\ D_x(0,y) = D_{0x}(0,y)\\
\psi(0,y) &= \psi_0(0,y),\ B_x(0,y) = B_{0x}(0,y)
\end{aligned}\tag{11}$$

and is

$$\varphi(0,y) = 0,\ \psi(0,y) = 0 \tag{12}$$

for the electromagnetically shorted case. In addition, the attenuation conditions are given by

$$\begin{aligned}
&w(x,y),\ \varphi(x,y),\ \psi(x,y)\to 0 \quad \text{for } x\to+\infty\\
&\varphi_0(x,y),\ \psi_0(x,y)\to 0 \quad \text{for } x\to-\infty
\end{aligned}\tag{13}$$

The propagation problem of the B–G wave in the functionally graded electro-magneto-elastic half-space (Fig. 1) can be solved using Eqs. (7) and (8) under conditions (10), (11), and (13) for the electromagnetically open case, and conditions (10), (12), and (13) for the electromagnetically shorted case.

3. Propagating wave solutions

3.1. Electromagnetically shorted case

The shear horizontal waves in the functionally graded piezoelectric materials have been investigated in a previous work (Qian et al., 2008). Hence, the solutions for the problem as a plane harmonic wave satisfying the governing Eq. (7) and the attenuation condition (13) are given by

$$\begin{aligned}
w &= A_1 e^{rx}\exp[ik(y-ct)]\\
\varphi &= (B_1 e^{sx}+m_1 A_1 e^{rx})\exp[ik(y-ct)]\\
\psi &= (C_1 e^{sx}+m_2 A_1 e^{rx})\exp[ik(y-ct)]
\end{aligned}\tag{14}$$

where A_1, B_1, and C_1 are arbitrary constants, $k(k = 2\pi/\lambda)$ is the wavenumber, λ is the wavelength, $i^2 = -1$, and c is the phase velocity of the B–G waves. $r = -\beta-\sqrt{\beta^2-k^2((c^2/c_{sh}^2)-1)}$ and $s = -\beta-\sqrt{\beta^2+k^2}$ with the bulk-shear-wave velocity in the homogeneous transversely isotropic electro-magneto-elastic material $c_{sh} = \sqrt{\bar{c}_{44}^0/\rho} = \sqrt{(c_{44}^0+m_1e_{15}^0+m_2q_{15}^0)/\rho}$, in which $m_1 = (e_{15}^0\mu_{11}^0-q_{15}^0\alpha_{11}^0)/(\varepsilon_{11}^0\mu_{11}^0-\alpha_{11}^{0\,2})$ and $m_2 = (\varepsilon_{11}^0q_{15}^0-e_{15}^0\alpha_{11}^0)/(\varepsilon_{11}^0\mu_{11}^0-\alpha_{11}^{0\,2})$.

The stress, electrical displacement, and magnetic induction components can be obtained by substituting Eq. (14) into Eq. (5), and the following equations are obtained:

$$\begin{aligned}
\sigma_{zx} &= e^{2\beta x}(r\bar{c}_{44}^0A_1e^{rx}+se_{15}^0B_1e^{sx}+sq_{15}^0C_1e^{sx})\exp[ik(y-ct)]\\
D_x &= e^{2\beta x}\left(-s\varepsilon_{11}^0B_1e^{sx}-s\alpha_{11}^0C_1e^{sx}\right)\exp[ik(y-ct)]\\
B_x &= e^{2\beta x}\left(-s\alpha_{11}^0B_1e^{sx}-s\mu_{11}^0C_1e^{sx}\right)\exp[ik(y-ct)]
\end{aligned}\tag{15}$$

Substitution of Eqs. (14) and (15) into Eqs. (10) and (12) yield the following homogeneous linear algebraic equations with respect to A_1, B_1, and C_1:

$$\begin{aligned}
r\bar{c}_{44}^0A_1+se_{15}^0B_1+sq_{15}^0C_1 &= 0\\
B_1+m_1A_1 &= 0\\
C_1+m_2A_1 &= 0
\end{aligned}\tag{16}$$

For nontrivial solutions, the determinant of the coefficient matrix has to vanish, which leads to the following dispersion relation that determines the wave velocity for the electromagnetically shorted case:

$$\frac{r}{s} = k_p^2 \tag{17}$$

where $k_p^2 = (m_1e_{15}^0+m_2q_{15}^0)/\bar{c}_{44}^0$ is the electro-magneto-elastic coupling coefficient, which is related only to the material parameters.

From Eq. (17), the following observations are made:

(1) The B–G waves in the functionally graded electro-magneto-elastic half-space are dispersive in the electrically shorted case because the phase velocity is related to the wavenumber, which is different from that in the homogenous material;

(2) If $\beta = 0$, the phase velocity of the B–G waves in a homogeneous transversely isotropic electro-magneto-elastic half-space can be obtained using the following equation:

$$c = c_{sh}\sqrt{1-k_p^4}; \tag{18}$$

(3) When the half-space is piezoelectric, that is, $q_{15}^0 = 0$ and $\alpha_{11}^0 = 0$, then $m_1 = e_{15}^0/\varepsilon_{11}^0$, $m_2 = 0$, $\bar{c}_{44}^0 = \bar{c}_{44}^e = c_{44}^0+e_{15}^{0\,2}/\varepsilon_{11}^0$, and $c_{sh}^e = \sqrt{\bar{c}_{44}^e/\rho}$, and k_p^2 is reduced to the electromechanical coupling constant $k_e^2 = (e_{15}^{0\,2}/\varepsilon_{11}^0)/\bar{c}_{44}^e$. Eq. (17) can then be written as

$$\frac{r}{s} = k_e^2 \tag{19}$$

which is exactly the same as the work of Qian et al. (2008). Furthermore, if the material is homogeneous, that is, $\beta = 0$, we can obtain $c = c_{sh}^e\sqrt{1-k_e^4}$, which is the velocity of the B–G waves in the homogenous transversely piezoelectric half-space for the electrically shorted case (Bleustein, 1968); and

(4) When the half-space is a functionally graded piezomagnetic, that is, $e_{15}^0 = 0$ and $\alpha_{11}^0 = 0$, then $m_1 = 0$, $m_2 = q_{15}^0/\mu_{11}^0$, $\bar{c}_{44}^0 = \bar{c}_{44}^m = c_{44}^0 + q_{15}^{0\,2}/\mu_{11}^0$, and $c_{sh}^m = \sqrt{\bar{c}_{44}^m/\rho}$, and k_p^2 is reduced to the electro-mechanical coupling constant $k_m^2 = (q_{15}^{0\,2}/\mu_{11}^0)/\bar{c}_{44}^m$. Eq. (17) can then be written as

$$\frac{r}{s} = k_m^2 \tag{20}$$

which, to the best of our knowledge, is a new outcome. Furthermore, if the material is homogeneous, that is, $\beta = 0$, we can obtain $c = c_{sh}^m\sqrt{1-k_m^4}$, which is the velocity of the B–G waves in the homogenous transversely piezomagnetic half-space for the electromagnetically shorted case (Wang et al., 2007).

3.2. Electromagnetically open case

The solutions governing Eq. (8) and satisfying the attenuation condition (13) are (Bleustein, 1968; Yang and Zhou, 2006)

$$\begin{aligned}\varphi_0 &= B_0 e^{kx}\exp[ik(y-ct)]\\ \psi_0 &= C_0 e^{kx}\exp[ik(y-ct)]\end{aligned} \tag{21}$$

The electrical displacement and magnetic induction components in air are

$$\begin{aligned}D_{x0} &= -\varepsilon_0 k B_0 e^{kx}\exp[ik(y-ct)]\\ B_{x0} &= -\mu_0 k C_0 e^{kx}\exp[ik(y-ct)]\end{aligned} \tag{22}$$

Substitution of Eqs. (14) and (15) into Eq. (11) yields

$$\begin{aligned}r\bar{c}_{44}^0 A_1 + s e_{15}^0 B_1 + s q_{15}^0 C_1 &= 0\\ B_1 + m_1 A_1 &= B_0\\ C_1 + m_2 A_1 &= C_0\\ \varepsilon_{11}^0 s B_1 + \alpha_{11}^0 s C_1 &= \varepsilon_0 k B_0\\ \alpha_{11}^0 s B_1 + \mu_{11}^0 s C_1 &= \mu_0 k C_0\end{aligned} \tag{23}$$

Using the same procedure as that in the electrically shorted case, we can obtain the corresponding phase velocity equation for the electromagnetically open case as follows:

$$\left(\varepsilon_{11}^0 s - \varepsilon_0 k + \frac{m_1\varepsilon_0 k s e_{15}^0}{r\bar{c}_{44}^0}\right)\left(\mu_{11}^0 s - \mu_0 k + \frac{m_2\mu_0 k s q_{15}^0}{r\bar{c}_{44}^0}\right) - s^2\left(\alpha_{11}^0 + \frac{m_1\varepsilon_0 k q_{15}^0}{r\bar{c}_{44}^0}\right)\left(\alpha_{11}^0 + \frac{m_2\mu_0 k e_{15}^0}{r\bar{c}_{44}^0}\right) = 0 \tag{24}$$

We make the following observations from Eq. (24):

(1) The B–G waves in the functionally graded electro-magneto-elastic half-space are dispersive in the electrically open case; these results are the same as those in the electrically shorted case;
(2) If $\beta = 0$, the phase velocity of the B–G waves in a homogeneous transversely isotropic electro-magneto-elastic half-space for the electromagnetically open case can be obtained as

$$c = c_{sh}\sqrt{1-\eta^2} \tag{25}$$

where $\eta = (m_1\varepsilon_0 e_{15}^0(\mu_{11}^0+\mu_0) + m_2\mu_0 q_{15}^0(\varepsilon_{11}^0+\varepsilon_0) + \alpha_{11}^0(e_{15}^0 m_2\mu_0 + q_{15}^0 m_1\varepsilon_0))/(\bar{c}_{44}^0[(\varepsilon_{11}^0+\varepsilon_0)(\mu_{11}^0+\mu_0) - \alpha_{11}^{0\,2}])$;

(3) When the half-space is piezoelectric, we can obtain the B–G waves in the functionally graded piezoelectric half-space using the same procedure as that in the electromagnetically shorted case. Thus,

$$\varepsilon_{11}^0 s - \varepsilon_0 k + \varepsilon_0 k\frac{s}{r}k_e^2 = 0, \tag{26}$$

which is the same as the work of Qian et al. (2008). Furthermore, if the material is homogeneous, that is, $\beta = 0$, we can obtain $c = c_{sh}^e\sqrt{1-k_e^4/(1+\varepsilon_{11}^0/\varepsilon_0)^2}$, which is the velocity of the B–G waves in the homogenous transversely piezoelectric half-space for the electrically open case (Bleustein, 1968); and

(4) Similarly, when the half-space is piezomagnetic, Eq. (17) can be written as

$$\mu_{11}^0 s - \mu_0 k + \mu_0 k\frac{s}{r}k_m^2 = 0 \tag{27}$$

which, to the best of our knowledge, is novel. Furthermore, if the material is homogeneous, that is, $\beta = 0$, we can obtain $c = c_{sh}^m\sqrt{1-k_m^4/(1+\mu_{11}^0/\mu_0)^2}$, which is the velocity of the B–G waves in the homogenous transversely piezomagnetic half-space for the electromagnetically open case (Wang et al., 2007).

4. Numerical simulations

A numerical example is presented to study the propagation behaviour of B–G waves in the functionally graded transversely isotropic electro-magneto-elastic half-space and to graphically show the effects of the gradient coefficient on the dispersion relation. The following parameters are used for the computations (Wei and Su, 2008): $c_{44}^0 = 43$ GPa, $e_{15}^0 = 11.6$ C/m^2, $q_{15}^0 = 550$ N/Am, $\rho^0 = 7500$ kg/m^3, $\varepsilon_{11}^0 = 11.2\times10^{-9}$ F/m, $\mu_{11}^0 = 5.0\times10^{-5}$ Ns^2/C^2, and $\alpha_{11}^0 = 5.0\times10^{-12}$ Ns/Vc. We assume that all material properties vary exponentially, as specified in Eq. (6). In addition, the dielectric and permeability constants in air are given by $\varepsilon_0 = 8.854\times10^{-12}$ F/m and $\mu_0 = 1.256\times10^{-6}$ Ns^2/C^2, respectively. Using the aforementioned parameters and Eqs. (18) and (25), we can obtain the velocities of the B–G waves in the homogeneous transversely electro-magneto-elastic half-space using $c_s = 2725.7$ m/s for the electromagnetically shorted case and $c_o = 2853.4$ m/s for the electromagnetically open case.

The variations in the patterns of the dimensionless phase velocity of the B–G waves c/c_o and c/c_s with the dimensionless wavenumber $|k/\beta|$ for the functionally graded electro-magneto-elastic substrate under both the electromagnetically open and shorted cases are shown in Fig. 2. The B–G waves are dispersive because of the interference of the functionally graded materials and are different from those of homogeneous materials. Meanwhile, Fig. 2 shows that the phase velocity starts at a lower value and monotonously increases to the velocity of the B–G waves as the dimensionless wavenumber $|k/\beta|$ increases. Furthermore, the effect of the gradient coefficient on the dispersion curve is more sensitive under the electromagnetically shorted condition than under the electromagnetically open condition; this result is similar to that obtained by Qian et al. (2008) when the half-space is piezoelectric. These two points validate the correctness accuracy of our calculation to some extent.

Fig. 3 shows the dimensionless phase velocity of the B–G waves with the wavenumber k for some selected β under the electromagnetically open and shorted conditions. For the

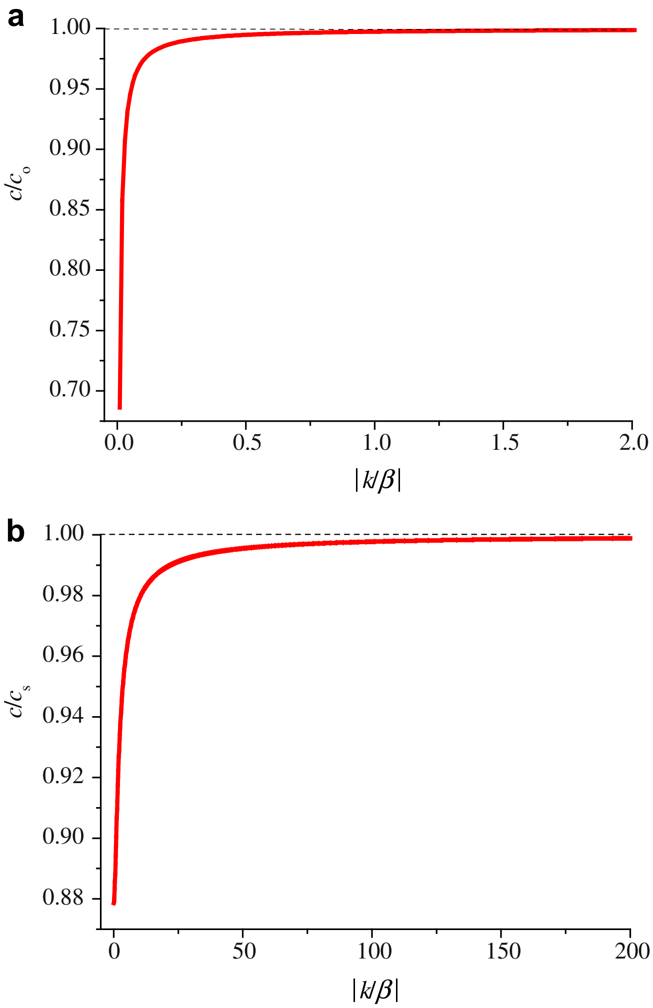

Fig. 2. Dimensionless phase velocity of the B–G waves with the dimensionless wavenumber $|k/\beta|$: (a) Electrically open case; (b) Electrically shorted case.

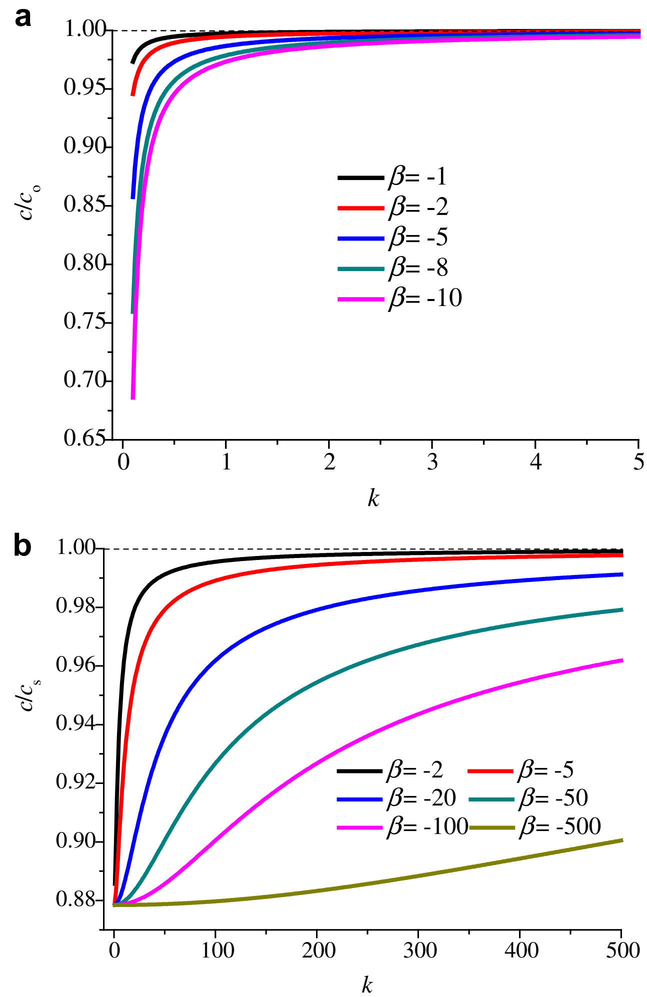

Fig. 3. Dimensionless phase velocity of the B–G waves with the wavenumber k for some selected exponential coefficient β: (a) Electrically open case; (b) Electrically shorted case.

electromagnetically open case, the initial value of the phase velocity decreases when the magnitude of the gradient coefficient is increased (Fig. 3a). On the other hand, for the electromagnetically shorted case, the initial value of the phase velocity remains constant and is lower than that of the B–G waves regardless of the gradient coefficient (Fig. 3b). As the wavenumber increases, the velocity of the B–G waves in the functionally graded half-space approaches the same values as that in the homogeneous substrate under the electromagnetically open and shorted cases. More importantly, the curve corresponding to $\beta = -500$ in Fig. 3b is significantly different from the other curves because of the large exponential coefficient.

The relationship between the dimensionless phase velocity of the B–G waves and the absolute value of the exponential coefficient β is quasilinear for some selected k under the electromagnetically open condition, whereas it is nonlinear in the electromagnetically shorted case (Fig. 4). These relationships may be used in practice. For example, the quasilinear relationship can be used to evaluate the value of the gradient coefficient based on the change in the phase velocity in the electromagnetically open case.

The group velocity c_g is introduced to demonstrate the dispersion relation more clearly. c_g expresses the rate at which the vibration energy is transported. It is defined as (Achenbach and Balogun, 2010)

$$c_g = \frac{d\omega}{dk} = c + k\frac{dc}{dk} \tag{28}$$

Fig. 5 shows the effect of the gradient coefficient on c_g. For the electromagnetically open case, the initial value of c_g decreases as the magnitude of the gradient coefficient increases (Fig. 5a). However, for the electromagnetically shorted case, the initial value of c_g remains constant and is lower than that of the B–G waves regardless of the gradient coefficient (Fig. 5b). These results are similar to the phase velocities of the B–G waves for the electromagnetically open and short cases. For the short waves, all phase and group velocities approach the values of the B–G waves under different conditions. On one hand, the anti-dispersion does not appear regardless of whether the material is functionally graded because of the larger phase velocity compared with c_g. On the other hand, c_g increases non-monotonously as the wavenumber increases; a minimum value that varies for the different gradient coefficients under the electromagnetically shorted condition exists, which is not the case for the electromagnetically open case.

For the engineering applications of SAW devices, a high electromechanical coupling factor of the wave is expected for the piezoelectric and electro-magneto-elastic materials. Similar to the electro-mechanical coupling factor of the piezoelectric material (Qian

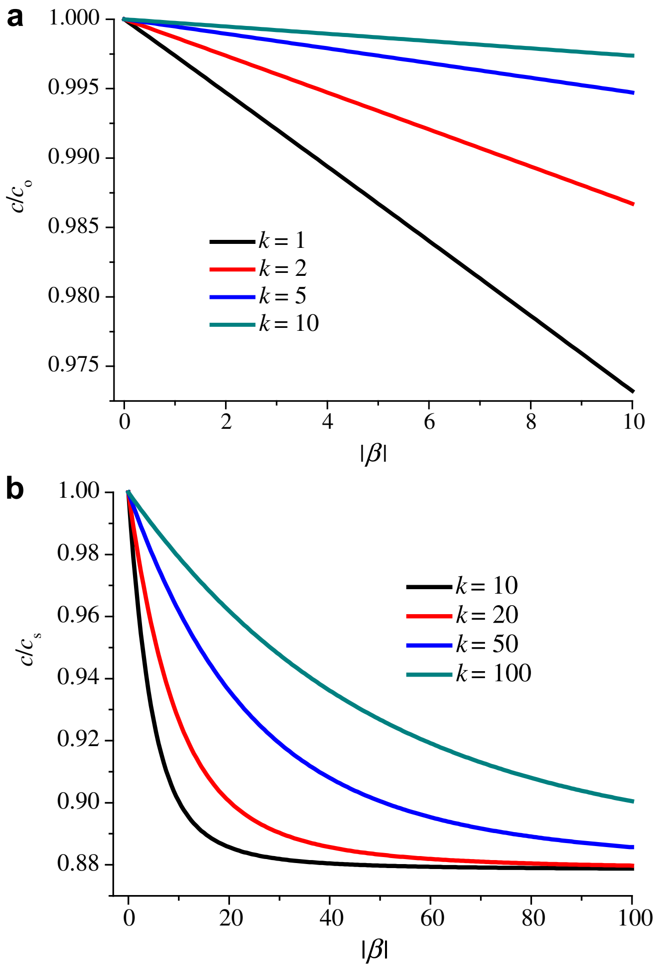

Fig. 4. Dimensionless phase velocity of the B–G waves with the absolute value of β for some selected k: (a) Electrically open case; (b) Electrically shorted case.

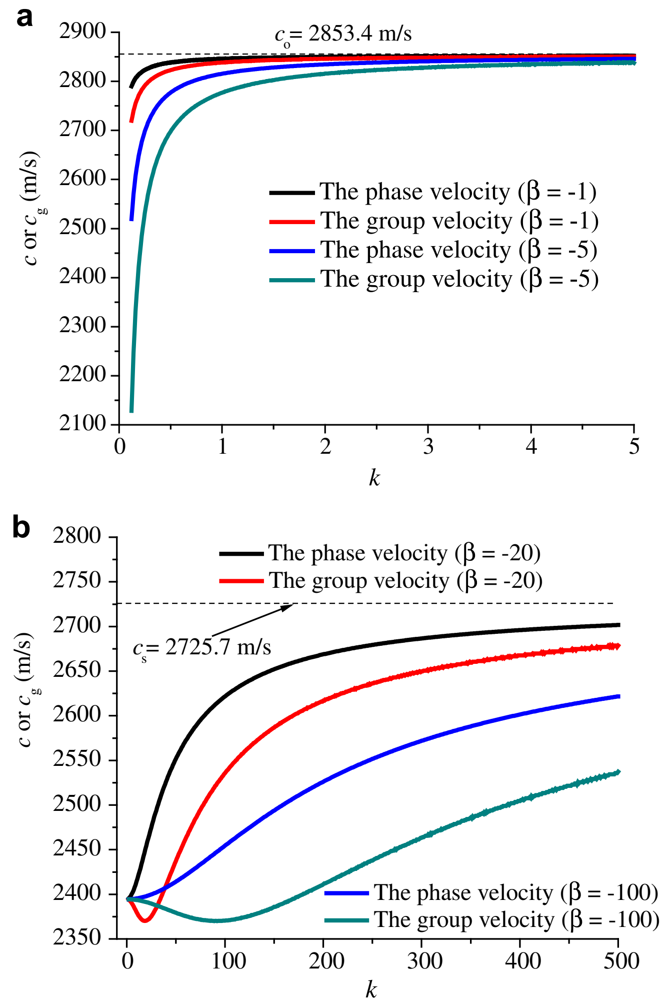

Fig. 5. Group velocities (c_g) of the B–G waves with k for some selected β: (a) Electrically open case; (b) Electrically shorted case.

et al., 2009, 2008, 2004), the electro-magneto-mechanical coupling factor K^2 of the electro-magneto-elastic materials for the surface waves is defined as follows:

$$K^2 = \frac{2(c_o - c_s)}{c_o} \tag{29}$$

where c_o and c_s are the phase velocities of B–G waves in the functionally graded electro-magneto-elastic substrate under the electromagnetically open and shorted conditions, respectively.

Fig. 6 shows the K^2 of the B–G waves with the wavenumber k for the homogeneous and functionally graded half-spaces. The gradient coefficient has a strong influence on K^2. The maximum value of K^2 for the B–G waves in the functionally graded substrate is approximately three times larger than that of the pure electro-magneto-elastic substrate, which is advantageous for the practical design of SAW devices because it would greatly increase the efficiency of the energy conversion in B–G wave propagation.

Given that the penetration depth of the B–G waves is up to 10–100 wavelength in the piezoelectric half-space (Jin et al., 2001; Qian et al., 2009, 2008), the practical application of the wave is limited to the microwave technology. For the transverse surface wave in the functionally graded electro-magneto-elastic substrate, the penetration depth can be computed from $A_1 e^{rx} = A_1 e^{-\beta-\sqrt{\beta^2-k^2(1-(c^2/c_{sh}^2))}x}$ in Eq. (14). Compared with the

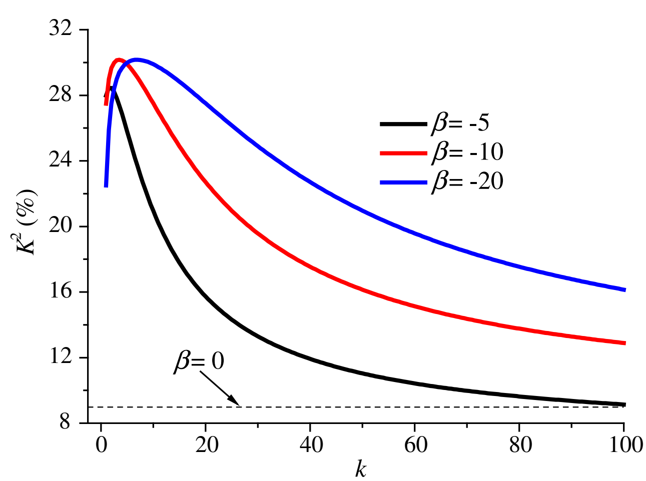

Fig. 6. Electro-magneto-mechanical coupling factor K^2 of the B–G waves with k for some selected β.

penetration depth of $A_1 e^{-k\sqrt{(1-c^2/c_{sh}^2)}x}$ in Eq. (9) for the pure electro-magneto-elastic half-space (Wang et al., 2007), the penetration depth of the B–G waves can be reduced because of the graded coefficient β.

5. Conclusions

In the current study, the effects of the graded coefficient on the propagation behaviour of the B–G waves in a functionally graded transversely isotropic electro-magneto-elastic half-space are investigated. The dispersion equations, which are reduced as special cases in literature to a few known elastic and quasistatic piezoelectric or piezomagnetic wave solutions, are analytically obtained from the three-dimensional equations of the linear electromagnetic static theory. A numerical example is given to illustrate the detailed effect of the graded factor on the propagation behaviour of the B–G waves in a structure (Fig. 1), the following conclusion remarks can be drawn:

(1) The B–G waves in the functionally graded electro-magneto-elastic half-space are dispersive under the electromagnetically open and shorted cases because of the inhomogeneity of the material;
(2) The effects of the graded coefficient on the B–G waves vary under different electromagnetic circumstances. For example, the effect of the gradient coefficient on the dispersion curve is more sensitive under the electromagnetically short condition than under the electromagnetically open condition. The relationship between the dimensionless phase velocity of the B–G waves and the absolute value of the exponential coefficient is quasilinear for some specific wavenumbers under the electromagnetically open condition, which can be used to evaluate the value of the gradient coefficient based on the change in the phase velocity; and
(3) The K^2 of the B–G waves can be improved by adopting the appropriate graded coefficient. The penetration depth of the B–G waves can also be reduced.

The dependence of the phase velocity on the presence of the functionally graded factor opens a new window in designing acoustic wave electro-magneto-elastic devices.

To adopt the material coefficients in exponentially change form make the analytical procedures in the current study feasible. Furthermore, the results obtained are meaningful for the design of high-performance surface acoustic wave devices. For practical application, much work needs to be carried out to coordinate the fabrication costs and benefits carefully even though some special material variation patterns can be achieved practically.

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